# "Which One is Special" Game & Teacher Guide

## Introduction to "Which One is Special"

Which One is Special is a general purpose activity based on the common game of deciding that one object is not like the others. For example, given a banana, apple, strawberry, and rose, the rose might be special because it's not a fruit. The banana might be special because it isn't red. Another reason the banana might be special is that it's much longer than it is wide.

## "Which One is Special" Teacher Guide

This guide offers suggestions on how to discuss these kinds of problems with students to promote their mathematical understanding. These kinds of problems are extremely flexible based on the objects being considered and how they differ from each other. Usually the objects will be similar in many ways and different in specific mathematically relevant ways. The overall goal is for students to try to think about those mathematical features when evaluating the objects to understand why any one of them is special compared to the others.

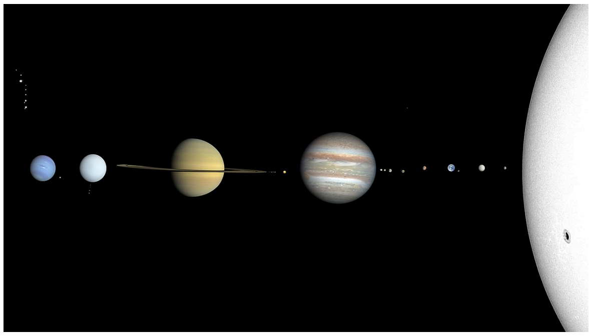
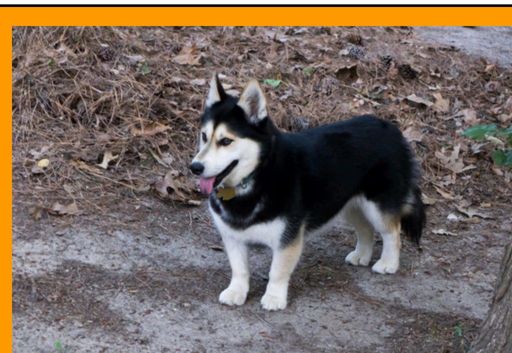
We hope this guide will help you guide discussions amongst and with students when they solve Which One is Special problems. Unlike many procedural forms of math, there will not be any single algorithm that solves problems and most problems will have multiple correct answers. The primary goal is for students to analyze the objects in many mathematical ways, then organize the information they uncover to help them differentiate one object from the others.

Of course, our suggestions are not the only way to use these problems, and we are sure you will find your own way to adapt these for your use. Hopefully this guide helps you get started!

## "Which One is Special" Examples

Finally, this guide uses examples drawn from the problems Enlearn provided for SAGA Education (formerly Spark Math and one of the EF+Math prototyping teams):

# "Which one is special" example [(Non)statistical questions]:

| | |
|---|---|
|  How many planets are in the solar system? |  What are the three breeds of pet dogs Americans think are the cutest? |
| What is the number of views on the most popular TikTok video? |  On average, how many fingers do humans have? |

## Learning goals [Common Core 6.SP.A.1]

The main goal is for students to understand the kinds of questions that statistics can answer. In general, a question is *statistical* if it requires repeated sampling and the answers could vary. "What is 2+2?" is not statistical because the answer is always 4 by definition. However, "What do 1st graders think 2+2 equals?" is statistical because we would need to go ask many 1st graders this question, and they would not necessarily all reply the same way.

It can be tricky to analyze whether a question is statistical or not, and there may not be a clear answer in some cases. On top of that, students may think that questions about facts or questions whose answers are numbers are statistical, but this is not generally true. Indeed, perhaps the most prevalent use of statistics nowadays is public polling, which is by definition about opinions and whose answers are usually names, not numbers. Just like we want students to understand when to use fractions or algebra to solve problems, we also want them to know when statistics is the right tool for answering real-world questions (see discussion questions).

## Prompts for helping students get started

**Orienting towards problem goals**

- We have four different questions here: how are they similar or different?
- What are we supposed to do with the highlighted question?

**Comparing ways in which questions can differ**

- How many answers are there to each question?
- Are the answers numbers, words, pictures, ideas, or something else?
- How would you find the answers to these questions? Would you look them up in a book, survey people off the street, look at some data from a computer, or something else?

**Reminding students about properties of statistical questions**

- Can anyone remember the definition of a statistical question?
- What is the difference between a fact and an opinion? Is that relevant here?
- If we asked the same question 10 years from now, could we get a different result?

## Possible student answers

This specific problem is meant to help students compare questions along the following axes:

- Statistical vs. non-statistical
- Fact vs. opinion
- Single answer vs. multiple answers
- Numerical vs. categorical or nominal (number vs. text answer)

Therefore, some *statistical* reasons as to why the cutest dog breed question is special:

- It's the only question with more than one answer - there are three answers.
- It's the only question with categorical answers - the answers are breed names.
    - The cutest dog breeds can't be sensibly plotted on a histogram or dot plot
- It's the only question that deals with opinions, not facts.

Other answers could include:

- The other pictures are mostly empty space, while this one is a photograph.
- It's the only question restricted to Americans specifically, as opposed to the whole world.
- It's the question whose answer could be worth a lot of money (i.e. pet advertisements).

## Organizational strategies for students to try

**Divide and conquer**

Try focusing on one question at a time and listing several facts about it, possibly covering up the other questions. For example, the TikTok question is about technology, the answer is a number, the answer probably changes each year, the answer is very large, etc.. The planet question is about space, involves huge objects, the answer is defined by scientists, etc..

After listing some facts about one of the questions, have students now consider an additional question as well. Ask whether the facts they found for the first question are also true for the second question. Follow up by asking for similarities and differences between the two questions, both including the lists they just generated and possibly new ideas.

**Chart organization**
Choose a fact or feature about the questions, like whether or not they have more than ten words. Then draw a chart and organize the questions according to whether they have more or fewer than ten words. For example:

| **Question contains fewer than ten words** | **Question contains more than ten words** |
|---|---|
| How many planets are in the solar system? | What are the three breeds of pet dogs Americans think are the cutest? |
| On average, how many fingers do humans have? | What is the number of views on the most popular TikTok video? |

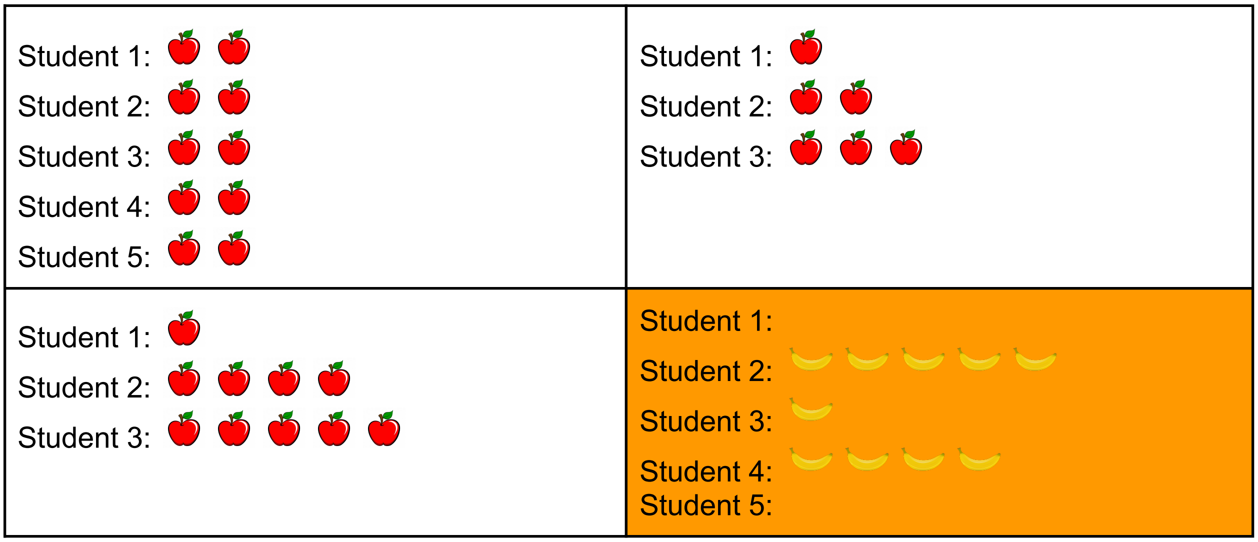
If such a chart results in three questions in one column and one by itself, then the one on its own is special. In this case, we have two questions in each column, so no question is special *based on whether it's longer than ten words or not*.

## Discussion questions after students have settled on some answers

These questions are designed to help students generalize their realizations from doing the activity. If students are able to answer these questions, they clearly have gained a strong understanding of what makes a question statistical.

- Compare the dog question to the planet question. At first glance the planet question might seem to be more mathematical because the answer is a number and a fact, while the dog question has word answers and about opinions. Why is the dog question statistical while the planet question is not?
- Would the dog question still be statistical if we asked what the principal thought were the three cutest dog breeds, rather than all Americans? Why or why not?
- Can students think of replacements to the existing questions that share their properties, such that replacing the questions would not affect the structure of the current problem? For example, "Americans' three cutest dog breeds" could be replaced with "Europeans' top ten cartoons." "Number of planets in the solar system" could be replaced with "Number of seconds in an hour."

# "Which one is special" example [Discrete distributions, mean]:




## Learning goals [Common Core 6.SP.B.5]

The main goal is for students to understand how statistical properties can be used to compare discrete distributions. Just as importantly, they should also recognize that information is lost when summarizing a distribution with a mean or median. For example, two distributions may have the same mean or median, yet still be quite different. This particular problem tries to help students use their intuition about *fairness* to recognize this idea, with real world and more advanced examples in the discussion questions.

This example is also chosen to help students think about two more subtle ideas:

1. "0" values in data can't be ignored like they can be with addition (they lower the average)
2. Distributions about different types of things can still be compared (apples and bananas)

## Prompts for helping students get started

**Orienting towards problem goals**

- What sticks out to you just looking at the different boxes?
- Can you come up with a story where the highlighted distribution might occur?

**Comparing ways in which distributions can differ**

- How many students are in each group?
- Which group would have the most fruit per student if the students were willing to share?
- Which group is it best to be student 3 in? What about student 1?
- What is the median number of fruit in each group? Repeat for min, max, range, mean.

**Drawing on intuition about fairness**

- How much fruit does the most fortunate student have in each group?
- How much fruit does the least fortunate student have in each group?
- Which group is most *fair*? Why?

## Possible student answers

This specific problem is meant to help students think about the following concepts:

- Central tendency (mean, median, mode)
- Descriptive statistics: min, max, range
- Different items can still be compared (both apples and bananas are fruit)
- 0s should not be ignored (adding a student with no fruit changes the distribution)

Some *statistical* reasons as to why the banana group is special:

- It has the lowest minimum number of fruit for a student: 0
- It has the most "unfair" distribution of fruit for some reasonable definitions of "unfair"
- It has the largest range of fruit per student (0 to 5)

Other answers could include:

- It's the only group with fruit other than an apple
- It's the only group whose data are not sorted in order from least to greatest fruit

## Organizational strategies for students to try

**Divide and conquer**

Start by having students list facts about each object. For example, the top left group has all students having the same number of apples, there are five students, and they have ten apples total. The bottom left group has all students having different numbers of apples, there are three students, and they have ten apples total. As students generate facts about one distribution, have them judge whether the same or different fact applies to the other distributions.

**Chart organization**

Choose a property of the distributions, and create a chart to keep track of which distributions share that property. For example:

| **2 fruit per student if they share** | **>2 fruit per student if they share** |
|---|---|
| Upper left: clearly all have 2 apples | Bottom left: More than 3 apples per student |
| Upper right: 3 gives 1 one apple | |
| Bottom right: 10 bananas for 5 people | |

This specific chart has three objects on one side and one on the other, so the distribution on the bottom left is special in that the students have more than 2 fruit on average. Of course, we are looking for reasons the bottom *right* group is special, but this is still interesting to note.

## Discussion questions after students have settled on some answers

How would you compare the situation between the bottom left and bottom right group? In particular, we normally can ignore 0 terms in other parts of math: $0+1+5+4+0 = 1+5+4$. Is it safe to ignore the "0" students in the bottom right, or does it change the classroom's situation?

Let’s say you’re a recruiter trying to convince a parent to send their student to the bottom right classroom. How might you try to “sell” the classroom? As a non-statistical example, you could say “This is the only classroom that has access to bananas!” Can you think of what statistical properties you would cite? What would you say if instead you wanted to *stop* parents from sending their students to that group?

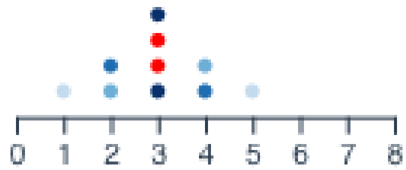
Online influences often sell courses or “business” opportunities by talking about their highest-earning followers. In what ways might this be misleading? What statistic(s) would be a fairer reflection of the chance that someone buying their course or investment will earn money?

In the United States, both of the following facts are true:

- The average household’s wealth is $748,800
- More than 50% of households have less than $122,000 in wealth.

How is this possible?

# “Which one is special” example [Distribution representations]:




| Value | Frequency |
|---|---|
| 1 | 1 |
| 2 | 2 |
| 3 | 4 |
| 4 | 2 |
| 5 | 1 |

Collected data:

1, 2, 2, 3, 3, 3, 3, 4, 4, 5

| Value | Frequency |
|---|---|
| 1 | 2 |
| 2 | 2 |
| 3 | 3 |
| 4 | 2 |
| 5 | 2 |

## Learning goals [Common Core 6.SP.B.4]

The main goal is for students to be able to convert between different ways of representing distributions: data lists, frequency tables, and dot plots. They should be able to recognize when distributions shown in different ways are the same or not, and be able to calculate statistics from different representations. A secondary goal is for students to think about the advantages and disadvantages of different representations, which will help them communicate more clearly with statistics in the future.

## Prompts for helping students get started

**Interpreting the distributions**

- What does each of these items *mean*?
- Imagine we made these distributions by asking students how many cookies they ate per day. Can you draw a picture of the students and how many cookies they ate?

**Practicing conversions from one representation to another**

- Try converting all of these distributions into a list of data points and compare them
- Try converting all of these distributions into a frequency table and compare them
- Try converting all of these distributions into a dot plot and compare them

**DIrect comparison by statistics**

- What is the mean/median/mode/min/max of each distribution?

## Possible student answers

This specific problem is meant to help students recognize distributions in different forms: dot plots, frequency tables, and a list of data points. The highlighted dot plot represents data that are the same as the data behind the top right frequency table and the lower left list of data points, so there are no *statistical* reasons the dot plot is special. Students may realize this when they start trying to compare the statistical properties of the different distributions.
With that said, some reasons as to why the dot plot is special:

- It's the only dot plot
- It's the only object with colors
- It's the only object without words
- It has the fewest number of characters
- It's the only object that contains the digit 0 (or 6, or 7, or 8)

## Organizational strategies for students to try

Usually, "Which one is special" problems can be tackled by trying to generate facts about the various objects and organizing the information in a table to identify why one object is special. However, since this problem is designed to get students to compare data represented in different forms, the most effective strategy will probably be to have students convert each object to the *same* form. If students are comfortable with dot plots and frequency tables, they can convert all objects into one of those forms. If they are still learning, try having them draw a picture representing the situation depicted in each object. You may need to provide a specific scenario since none of the objects are labeled (i.e. the data represent how many cookies each student ate in the class).

Once they convert all objects to the same form, it should immediately become apparent that three of the four objects represent the same distribution; since the highlighted distribution is one of those three, students should realize that there is no statistical reason that the highlighted distribution is special. They can then focus on purely visual properties.

## Discussion questions after students have settled on some answers

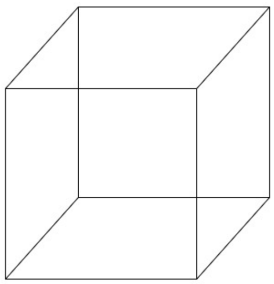
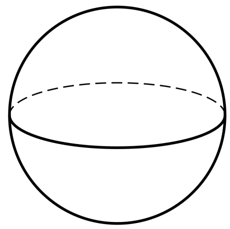
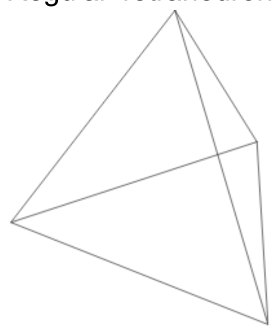
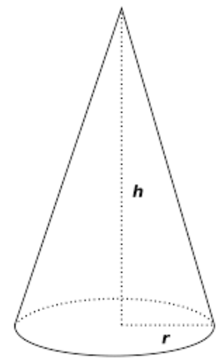
Which of the different ways of viewing the data do you *like* the most and why?

If you needed to calculate the mode, which format would you want your data to be in?

Which format would be best if you wanted to describe how many children are in each household in the United States? How long would a frequency table be? How tall would a dot plot be? How long would a list of data be?

Say you wanted to represent the cost of every item for sale on Amazon. Which format would you want to use, if any? Why do you like or dislike each possible format?

# Explore “Which One is Special” - Geometry

| Cube | Sphere | Regular Tetrahedron | Cone |
|---|---|---|---|
|  |  |  |  |

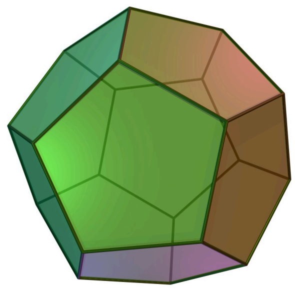
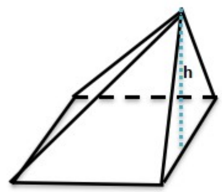
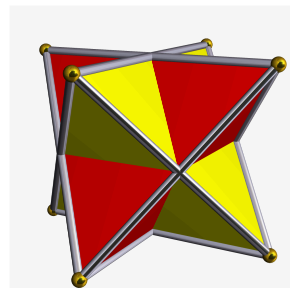
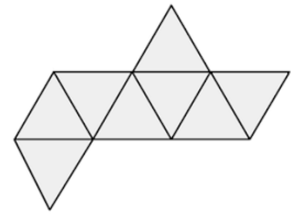
Something special about each item:

**Cube**:
- Only objects whose volume formula lacks a ⅓ term
- Only object containing a square (quadrilateral) of any kind

**Sphere**:
- Bolder lines than the other diagrams
- Only object that can roll across a surface
- Only object with no edges
- Only object with no vertices

**Regular tetrahedron**:
- Only object with triangular faces

**Cone**:
- Only diagram with labels
- Only object with two degrees of freedom (radius of base and height)
- Only object with a curved edge
- Only object whose surface area formula contains a square root
- Only object with a single vertex

| | | | |
|---|---|---|---|
|  |   |  |  |

Most of the answers here rely on recognizing that the last item is a flattened representation of a regular octahedron.

Something special about each item:

**Dodecahedron**:
- Only object with non-triangular faces
  - Only object whose surface area can't be derived from triangle area formulae
- Only object whose volume can't be derived from pyramid volume formulae
- Only object whose surface area (and volume) can't be derived from class material (?)

**Irregular pyramid**:
- Only irregular object, i.e. not all edges are equal length
  - Alternative, only object requiring more than 1 edge length to define
- Only object with a labeled height
- Only object with a dotted line for perspective
- Only object with a face that contains an oblique angle
- Has the fewest faces

**Stellated octahedron**:
- Only non-convex polyhedron (students may refer to it as "pointy" in some sense)
- Only object with extra points denoting vertices
- Only object with extra thick lines for edges
- Has the most faces
- Note: Surprisingly, this shape *can* be folded flat like in the last image (i.e. it has a net)

**Flattened octahedron**:
- Only object that is two dimensional
  - Only shape with no volume
  - Only object requiring folding to assemble a polyhedron
- Only object where a non-isometric shape would result in the same polyhydron, i.e. there are other ways the triangles could be arranged resulting in an octahedron after folding

| | | | |
|---|---|---|---|
| $\frac{1}{3}\pi a^2 b$ | lw+wh+lh | ⅓Bh | $\frac{1}{8}\pi d^2 h$ |

**Opening question:** Draw diagrams of what these geometric formulas might represent.
**Hint**: a, b, l, w, h, d are all in units of length, while B is in units of $length^2$

Something special about each item:

**$\frac{1}{3}\pi r^3$:**
- This could be the volume of a quarter of a spheroid
- Only object lacking an h term

**lw+wh+lh:**
- This could be half the surface area of a rectangular prism
- Only formula representing an area, not a volume
- Only formula that’s degree 2, not 3
- Only formula with addition symbols
- Only formula lacking a fraction term
- Only object that can’t represent a curved shape (depending on drawn diagrams)
    - Equivalently, only object that can’t roll or rock
    - ⅓Bh could have curves if B is referring to a circle, i.e. the shape is a cone

**⅓lwh:**
- This could be the volume of any kind of cone or pyramid
- Only formula measuring a “complete” object rather than part of an object
- Only formula (debatably) generalizing across several different “kinds” of shapes
- Only formula with a capital variable, B
- Only formula with a variable in units of area, not length

**$\frac{1}{4}\pi d^2 h$**:
- This could be the volume of half a cylinder
- Only formula implied to be given in nonstandard units (diameter and not radius)