The Swap Puzzle

Introduction to the Swap Puzzle

The Swap Puzzle is designed to promote mathematical discourse and support student's executive function skills. It involves a problem-solving task that requires students to consider and compare multiple possible swaps of characters (e.g., digits, operators) in a math problem to get the largest possible number.

Key attributes of the Swap Puzzle:

- Provide space for students to explore mathematical concepts and ideas as they engage in sense making and problem solving
- Encourages (requires) students mathematical talk
- Easily adaptable for different mathematical topics
- Accessible for students with varied levels of proficiency
- Engaging for students- there is always something to try to move forward in their thinking and work

Click here to introduce yourself to the Swap Puzzle.

How does the Swap Puzzle support the development of executive function skills?

Executive function skills (EFs) are a set of higher-level skills that humans use to focus, plan, and manage multiple tasks. EF skills are a bit like the management of a company: they don't do work on the ground, but they help coordinate work towards a larger goal. We will be focusing on three core EF skills that research has shown have major positive associations with math learning over the long run: cognitive flexibility, working memory, and inhibitory control.

EFs as a general life skill is invaluable for helping people decide what, when, and how to accomplish goals. People with executive *dys*function have trouble planning for the future, problem solving, and refraining from poor but attractive decisions. In academic contexts, EF skills are necessary for any problem that is not solved by rote memory, with greater EF required for particularly tricky or non-obvious problems.

The swap puzzle primarily trains *cognitive flexibility*, the ability to think about the same problem from multiple angles. Most swap puzzles don't have a single obvious solution. Students must therefore consider several different swaps and compare them, then develop strategies for new swaps to test. This "try, test, learn" loop is practically the definition of cognitive flexibility.

To a lesser degree, the swap puzzle also trains *working memory* and *inhibitory control*. Working memory is the ability to hold several pieces of information in immediate memory at the same time. Swap puzzles require working memory to compute possible swaps without breaking out a calculator or piece of paper. Inhibitory control is the ability to avoid doing something even though it seems attractive at first glance. Puzzles with operators and decimal points tend to require inhibitory control because clever swaps of operators can produce better swaps than more obvious “digit” swaps.

How do we help students develop executive function skills?

This is an open research question. With that said, we have some suggestions:

Cognitive flexibility

A cognitively *inflexible* student would choose a single strategy, execute it, and refuse to consider alternatives even on problems where their old strategy clearly doesn’t work. Therefore, to train cognitive *flexibility*, we should:

- Encourage students to try many swaps (even if some of them result in smaller values), analyze what they’ve learned from trying swaps, and generate new swaps. For example, students might realize that exponents tend to contribute more to the size of an expression than additions, or that operations that normally make numbers larger can do the opposite when negative numbers are involved. It’s more important for students to try many swaps rather than find the best one.
- Students should try to apply strategies they learned from previous problems to new problems. If these strategies don’t work, they should clarify or learn new ones.
- For a true challenge, students should create their own problems or write hints for students solving future problems. A good hint would explain what the hint is, when it applies, and why it works (potentially with examples). For example: “Have you thought about trying to swap a larger number into the exponent? This works best in puzzles with a large positive base and small exponent. If you make the exponent bigger, it multiplies the base (which is big) several more times, which makes the number a lot bigger compared to addition or moving decimal a point.”

Working memory

A student with low working memory will not be able to do math in their head, plan several steps into the future, or remember simultaneous ideas. Working memory is the most difficult of the fundamental EF skills to directly train. That being said, students can learn strategies to *aid* their working memory, even if their working memory itself stays the same:

- Have students show their work when calculating the value of a swap, so they aren’t doing as much work in their heads.
- Ask students to keep a list or table of the swaps they have already tried, so they don’t need to remember their previous attempts.

- Have students organize their swaps, either grouped by strategy or on a number line by value, so they can easily remember what they were trying to do and quickly remember the best swap they've tried so far.

Inhibitory control

A student lacking inhibitory control would choose the most obvious answer immediately and move on to the next problem without checking their work. To train inhibitory control, we should:

- Ask students to show or prove that one swap is larger than another. This may require drawings, visualizations, or manipulatives like number lines or place ten blocks.
- Be willing to try swaps or strategies that didn't work before, but might now.
- Think about what the "strangest" or "least obvious" swap might be, even if it doesn't make the number larger.

Conclusion

Executive function skills (including cognitive flexibility, working memory, and inhibitory control) are important in life generally and mathematics specifically. We have designed the swap puzzle specifically to help introduce and practice these executive function skills, as there are many strategies students can use to solve a puzzle but there is no single strategy that is best for all puzzles. With that said, executive function is useful far beyond this single activity. We hope that after using the swap puzzle, teachers will go on to develop their own ways of encouraging students to learn about and develop their executive function skills.

The Swap Puzzle: A novel strategy for supporting mathematical discourse and executive function in high school classrooms

Yesmi Rios

Enlearn

What is a swap puzzle?

A novel approach for designing tasks

Provide space for students to explore mathematical concepts and ideas as they engage in sense making and problem solving

Encourages (requires) students mathematical talk

Easily adaptable for different mathematical topics

Accessible for students with varied levels of proficiency

Engaging for students- there is always something to try to move forward in their thinking and work

Now let's see one and try it out!

The swap puzzle: Try it out

Swap two characters in the number below.
You can swap anything, even operators or decimals.
Your goal is to get the largest number.

Swap Puzzle 1: 617×6^{1}

The swap puzzle: Let's discuss

What did you try first? Why?

How did you know what to do?

What number did you make?

How do you know that it is the largest number?

If you didn't make the largest number on your first try, what information would help you with the next puzzle?

Swap two characters in the number below.
You can swap anything, even operators or decimals.
Your goal is to get the largest number.

Swap Puzzle 1: 617×6^{1}

The swap puzzle: Let's try more!

What we learned from the last puzzle is that swapping the largest possible digit into the exponent can make a number much larger.

Now let's try these two puzzles, which have a similar structure to the previous puzzle.

Swap Puzzle 2: $1^{4} \times 9$

Swap Puzzle 3: 83×2^{2}

The swap puzzle

Here are the best swaps

Swap Puzzle 2: $1^{4} \times 9 \rightarrow 9^{4} \times 1$

Swap Puzzle 3: $83 \times 2^{2} \rightarrow 8 \times 3^{2^{2}}$

What swaps did you try? Which one do you like best and why? What did you notice about exponents, multiplication, or place value?

What's special about the swap puzzle?

Simplicity and flexibility: The novel approach for Swap Puzzles

Easy to explain: rules never change

Broadly applicable: works on any expression with a value

Accessible, especially for ELLs (no English proficiency requirement, which offers many entry points for students)

No blank pages: students can always try a new swap

- Students are less likely to get stuck in ways that prevent them from moving forward , they can always try a new strategy

Creates opportunity for rich mathematical discourse

- Students discuss their approach, they discuss their strategies, they argue about different ways to approach and work on different puzzles, make connections among mathematical ideas and topics. Etc.

Simplicity and flexibility

14×2^3

31.45

$\frac{931}{-2}$

Number sense and operational mastery

$14 \times 2^3 = 112$

A. $24 \times 1^3 = 24$
B. $34 \times 2^1 = 68$
C. $14 \times 3^2 = 126$
D. $41 \times 2^3 = 328$
E. $43 \times 2^4 = 688$
F. $12 \times 4^3 = 768$
G. $1 \times 42^3 = 74088$

Number sense and operational mastery

- Number sense
  - Constantly comparing similar expressions
  - Learn how each part contributes to the overall value
- Operational mastery
  - One problem gives many chances to practice operations
  - Learn how changing one part of an operation affects its value
  - Compare relative strengths of different operations
- Explore the language of math
  - -91 + -3 => - -1 +93?
  - (-3^2) => $(-3)^2$
  - -1^{23} => 21^{-3}

How can Swap Puzzles help with cognitive flexibility?

- What is cognitive flexibility?
  - Definition 1: Solving the same problem in many ways
  - Definition 2: Being willing to let go of old ideas with new evidence
  - One of the fundamental *executive function* skills
- How does the swap puzzle support cognitive flexibility?
  - Easy to try different solutions: try as many swap(s) as you want
  - Many ways to succeed: usually, several swaps will be larger
  - Operator swaps require thinking outside the box ($14 \times 2^3 \Rightarrow 1 \times 42^3$)
- Many ways to make puzzles harder
  - More complicated expressions
  - "Write a hint for your friends who will solve similar puzzles"
    - Can share authority in math classrooms with students because students can become "authors" and "puzzle creators" by generating new puzzles and/ or hints
  - "Find a puzzle that looks the similar but needs a new strategy" ($4^3 \times 9 \Rightarrow 1^3 \times 9$)

Swap puzzles promote mathematical discourse

- Many swaps => many ideas => many ways to explain
  - "I moved a bigger number to the hundred's place because…"
  - "I swapped the 9 into the exponent because…"
  - "The denominator is making the whole fraction negative, so I…"
- Promotes discussion with *mathematical* focus
  - No simple algorithm solves all problems
  - Avoids explanations like: "I swapped ____ because that's how we learned to do it in class"
  - Promotes explanations like: "My best swap was making the exponent bigger because___"
- Encourage creativity and pro-social problem solving
  - Create puzzles for others to solve
  - Write hints for friends for similar puzzles
  - Explain different parts of the expression