Gaining Information by Modifying Problems: A Resource to Support English Language Learners

Introduction

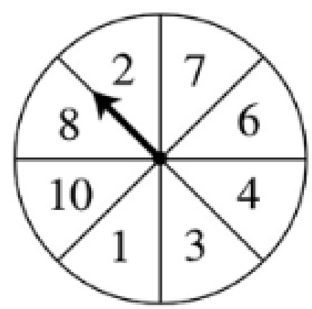
Students learning English may struggle when solving math problems due to misinterpretation of key vocabulary. This resource introduces a method of modifying problems as a powerful feedback strategy for educators. Instead of giving direct hints or answers, this method involves presenting students with variations or components of the original problem. This approach provides educators with the opportunity to gain valuable information about the student's specific point of difficulty (e.g., a vocabulary misunderstanding versus a conceptual error) while maintaining mathematical rigor and allowing students to demonstrate their knowledge.

Language learners and math

English language learners (ELLs) generally perform worse on math tests than native English speakers with the same level of math knowledge. To understand why, consider this example from a research study where less-common vocabulary has been stripped out:

To win a game, T----- must ---- -- ---- number on a --------- -------- -- –- one ----- ------.

Are T-----'s chances of --------- -- ---- number ------, likely, unlikely, or -----------?



A. ------
B. likely
C. unlikely
D. -----------

- Can you infer what the missing answers might be?
- Based on the image, what kind of game might be played?
- What might be the winning condition of the game? How sure are you?

When the researchers spoke with ELL students who answered this problem incorrectly, most ELL students had answered "C. unlikely" and circled the number 1 on the spinner. With this much missing information, it is difficult to figure out the win condition. Students saw the word "one" and guessed that the win condition was spinning a 1 on the spinner, in which case the answer was indeed "unlikely."

If we insert back some of the missing words, we get:

To win a game, T----- must <u>spin</u> -- <u>even</u> number on a --------- -------- -- <u>the</u> one <u>shown</u> ------.

In regular language, “even” can often be ignored in favor of understanding verbs and nouns. Unfortunately, in math it has a critical alternative meaning. In this example, ignoring “even” led to a situation where ELL students inferred the meaning of most of the problem, figured out what the missing answer options meant, and performed correct probabilistic reasoning, yet ended up with the wrong answer.

Modifying problems as feedback

This particular sequence of events happens all the time, to both ELL students and native English speakers. A misinterpretation of a key phrase or what the question is asking, or even one incorrect step somewhere in the middle of a problem, can lead to (possibly wildly) incorrect answers. In a perfect world, we would be able to give students credit for all the math they did correctly, and gain information about what specifically went wrong so we can give them the appropriate explanation or practice. In the previous spinner example, the problem is about probability - but a student who got the wrong answer because they missed the word “even” does not need to practice probability at all, but rather mathematical vocabulary.

One type of feedback strategy we think educators should add to their toolbox is to give modifications of problems to students first, rather than direct hints or answers. These modifications should either encourage students to use problem solving strategies they can use on future problems, or reveal more information to the educator about their understanding of the problem. These types of problem modifications vary wildly, but some examples include:

Ask students to record intermediate steps

- Which numbers does Tamika want to spin on the spinner and why?

Identify relevant mathematical vocabulary

- Underline the important math words in the problem. What do they mean?
- Which words do you think could be important, but you don’t recognize?

Demonstrate understanding of a problem component

- Circle the even numbers in the following list: 5, 6, 8, 3, 1, 2, 0
- Draw some possible outcomes of playing this spinner game

Solve a simplified version of the problem

- Let’s say Tamika wins by landing on black. Are Tamika’s chances of winning certain, likely, unlikely, or impossible?

Advantages of problem modification instead of hints

Keeps the original mathematical rigor: Say a student comes to a realization about how to solve the original problem while solving the modified problem. None of these problem modifications actually gave away any part of the answer, so we would be happy to give the student full credit for the original problem. This gives the student a second try at the "full-powered" original problem while still giving them an idea about new strategies to try.

Allows students to demonstrate their knowledge: It is almost never the case that students know absolutely nothing about a problem. It is far more common that students can do *most* of a problem, but are missing one or two pieces of information that would allow them to finish. These modified problems allow students to explicitly succeed at the parts of the task they know how to do.

Helps educators understand what went wrong: If a student is able to reason about the "black spinner" suggestion above but not the original, then we know the student understands what spinner games are, the meaning of probability words like "impossible" or "unlikely," and the general concept of probability. This means the problem the student was having was likely about the win conditions, or something about the numbers themselves. This is valuable information that helps direct the next round of feedback.

Helps teach students alternative strategies: Almost any valid strategy for solving problems can be given as a modification prompt. In the above examples, having students circle the winning numbers is a good strategy that falls under "recording your work to refer back to it later." As another example, in the equation $2x+4=10$, asking students to demonstrate some values of x that *do not* solve the equation is helping students learn the guess and check strategy. Or in the same example, asking if the value of x could be negative helps students learn the strategy of "narrowing down possible answers."

A new classroom culture for grading

One of the most empowering grading strategies I've experienced in a classroom is from a high school physics class. We often had problems with diagrams involving force vectors. If we didn't know how to solve the trigonometry to get a precise answer, the teacher would give partial credit for drawing the most accurate diagram possible and measuring the answer with a ruler; this is because in physics and engineering, the most important question is usually how to model a problem and understand "roughly" what the answer is, rather than getting a precise answer to several decimal points. This way we could still show that we knew what the problem was asking, we just lacked a specific mathematical skill to arrive at a closed-form solution.

We think educators should *want* to give their students as much deserved credit as possible. Imagine a classroom where if a student knows they are not getting the right answer to a problem, they know they can earn partial credit by creating and solving easier variations or parts of the problem. First, this is just nicer and more humanizing: imagine how much better performing a show is in front of a supportive audience than a hostile one. But it's also a powerful incentive for students to break down problems into smaller parts so they can gain credit for showing they know how to do individual parts, while simultaneously learning and practicing high-level strategic skills that mathematicians and engineers use every day. This is the kind of rigorous math all of us should aspire to do.

Why did Enlearn make a PD course about problem modification?

Enlearn specializes in "clever" technological solutions to educational problems that normally require tutors or educators. Many of our prototypes are puzzles or require lateral thinking. A common thread we have found for these prototypes is that users often can do parts of the problem but might misunderstand one of the rules or even our instructions. Unfortunately, these simple mistakes mask their true level of knowledge from our software. We are therefore very interested in feedback mechanisms that give the computer more information about what specific problem the user had.

This situation mirrors the difficulty that ELL students have with seemingly "straightforward" problems. In the spinner example above, for an ELL student, the problem *is* a puzzle because the student might be missing a lot of information about the problem for language reasons, as opposed to mathematical ones, and would need to do a lot of inference to figure out the missing pieces. In our own work we had been finding success with the computer modifying the original problem or puzzle to give the student another chance to demonstrate their mathematical ability, without giving away any part of the answer. Thus we thought that a similar process could be useful for teachers to add to their toolbox when responding to their own students, especially ELLs, and decided to create a PD course based on this idea. Plus, we would love to see the kinds of problem modification strategies that educators use with their students, and find ways to program these strategies back into our own prototypes!

Appendix: problem modification with procedural problems

At first glance, it might seem it would be difficult to apply this strategy to procedural problems. However, that's not necessarily the case. Consider the following linear equation problem:

Solve for x: $2x+4=10$

Test a solution
- Test any possible solution for x and show if it does or doesn't solve the problem
- Find the answer by guessing and checking

Narrow down possible solutions
- Is it possible for x to be less than 0? Why or why not?
- Is it possible for x to be more than 10? Why or why not?

Demonstrate understanding of a problem component:

- In your own words, what does it mean to “solve for x?”
- What does this “=” mean in this problem?
- What does the “x” mean in this problem?

Recognize similar problems

- How is this problem different from or similar to 10=4+2x?
- How is this problem different from or similar to 2y+4=10?

Solve parts of the problem

- Solve the problem 2x=10
- Solve the problem x+4=10

Model the problem

- Draw a picture that might represent this equation
- Write a word problem that might represent this equation

In general, we try to break down everything a student needs to know to solve a problem. This usually involves a lot of smaller skills, some of which might not be obvious. In the equation example, students need to know what variables are and what the “=” sign means. They also need to understand what it means to add and multiply, add or subtract from both sides, multiply or divide both sides, check a possible solution, and recognize how this kind of math might occur in the real world. Then, for each of these skills, we think about what related question we could ask that would give students the opportunity to demonstrate that skill and help them build intuition about the original problem.

As an example of a great outcome, imagine a student who knows what an equation is but has forgotten the procedure to solve equations. In response, we ask them not to solve the equation, but to answer the question “Is it possible for x to be less than 0? Why or why not?” The student might try to answer the question by guessing and checking a negative answer, such as -1. 2*-1+4 is clearly less than 10, so it doesn’t work. Then the student might try a more negative number, like -10. 2*-10+4 is even smaller, so it’s even worse as an answer; it seems that no negative answer is going to solve this equation.

But the student might notice as they try solutions that when they multiply 2*x, they will always add 4 to the answer and want to get 10. The problem with negative values of x is 2*x is always negative, so adding 4 is not enough to get to 10. If only they could end up with 6 after multiplying 2*x, then they would have the solution. At this point, they might remember that 2*3=6, and guess x=3 as the solution. In this scenario, the student was able to use the information they learned from solving an easier variation of the problem to find the correct answer; even better, we never actually told them any part of the solution, so in fact they solved the full problem on their own and deserve full credit. And as an added bonus, they might even remember that they can use a similar strategy of asking whether the answer is positive or negative for future problems.

If you’re interested in learning more, see the Enlearn resource, "Reimagining Math Assessments to Engage and Empower English Language Learner Students," on the EF+Math Research & Tools page.