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STUDENT INTERESTS AND TEACHER COMMITMENTS: NEGOTIATING ITERATIVE CO-DESIGN OF AN INNOVATIVE MATHEMATICS CURRICULUM INTEGRATING EXECUTIVE FUNCTION PROCESSES

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Over the past three years, a team of researchers, school administrators, and teachers in the United States have been co-designing Spark Math, a middle grades mathematics curriculum that integrates executive function processes, mathematics content, and problem contexts informed by students' interests. Executive function (EF) processes – e.g., working memory, inhibitory control, flexible thinking – have been introduced and developed alongside mathematical activities. Through a curriculum co-design process, we explored the tensions and synergies between teacher perception of fit, recognition of student interests, universal design for learning, EF processes, and mathematics content goals. This paper discusses how we engaged in this iterative co-design process and managed the tensions that emerged.

RATIONALE

Our project team sought to develop middle grades mathematics curriculum that could exemplify productive ways to integrate executive function (EF) processes with mathematical activities in regular classroom settings. This project, Spark Math, engaged teachers as co-design partners who worked in public schools in the United States serving a high proportion of low-income students and minoritized students who are underrepresented in mathematics. Teachers were involved as co-designers from the first discussion of activities and instructional sequences, through the piloting of initial versions of the curriculum, to the implementation of curricular revisions based on teacher feedback. This paper describes the co-design process, the challenges and tensions that emerged, and how these tensions were managed.

THEORETICAL LENS

Design-based research is an approach for understanding relationships and interactions among classroom practices, instructional resources, and theories of how students learn (Design-Based Research Collaborative, 2003). When pursuing educational innovation in mathematics classrooms, some researchers have argued that teachers should be acknowledged as partners and collaborators in the design process. This contrasts with more conventional cases of curriculum development in mathematics education in the United States, in which teams of mathematics educators and mathematicians produce instructional materials that are then piloted by mathematics teachers, most of whom were not part of the design process (cf. Hirsch, 2007).

Co-design as a negotiated process

By partnering with teachers earlier in the curriculum design process, the resulting instructional resources may be more readily adopted by teachers and reflect their perspectives on what mathematics is, how task design prompts student responses (e.g., recall, sense making, problem solving), and their understanding of how students learn mathematics. An underlying tension with co-design – which has been described by Roschelle et al. (2006) as “a collaborative approach to developing innovations that ‘fit’ into real classroom contexts” (p. 606) – is that the purpose of curricular innovation often is to disrupt status quo norms and practices. On the other hand, without teacher input mathematics curricula can become so “innovative” that only the most exceptional teachers understand how to use it effectively. Of course, effectiveness is in the eye of the beholder. Curriculum designers and teachers may have differing opinions regarding important outcomes such as performance on classroom assessments, shifts in the quality of classroom discourse, improved student engagement and participation, etc. It is important, therefore, for designers and educators to negotiate and revisit expected outcomes through iterative phases of co-design.

One of the primary goals of the Spark Math project team was to design instructional materials that integrated, enhanced, and developed EF processes in mathematics classrooms. However, we also endeavored to design the instructional resources in such a way that most middle-grade mathematics teachers (including novice teachers) could productively enact the activities in their classrooms. To accomplish this, we engaged teachers who planned to use the curriculum as co-designers from the beginning of the project – i.e., before any instructional materials were fully developed. We also invited co-design teachers to ask their students what they liked and didn’t like about mathematics, solicit information about problem contexts that interested them, and have students share advice they might give if they could design a game. All of which was considered while developing the curriculum.

Core Executive Function Processes

With respect to mathematics education, EF processes are understood as cognitive activities that contribute to and are influenced by tacit and explicit socio-mathematical norms in schools (Yackel & Cobb, 1996) and norms among peers, families, and cultures that influence how students experience mathematics education. As such, EF processes are more often studied as individual cognitive processes. When considering how our instructional materials would be used by groups of students and teachers in classrooms, we prioritized sociocultural perspectives on learning (Cobb, 1994; Vygotsky, 2012) as a theoretical lens for interpreting the role of EFs in supporting student learning.

Arguments for giving greater attention to EF processes in education often reference EF intervention studies that demonstrate greater improvements among children in lower-income contexts (Diamond & Lee, 2011). We identified the three core EFs identified by Diamond and Lee (2011) – i.e., cognitive flexibility, inhibition, and working memory – as processes that could be readily associated with goals for mathematics education. For example, student engagement with mathematical activities requires interpreting and using related mathematical representations, persistence in problem solving, and recall and management of information (Kharitonova & Munakata, 2011; Ashcraft & Kirk, 2001).

Cognitive flexibility is engaged when switching between multiple tasks, interpreting information in different ways, and shifting to different strategies as needed. Approaches for developing cognitive flexibility include studies involving solving puzzles, playing games, and even the use of puns or jokes since they often include multiple meanings. In mathematics education, tasks open to multiple solution strategies or requiring the use of multiple representations engage learners’ cognitive flexibility. Classroom discourse includes listening to, observing, and comparing solution strategies. Comparing one’s solution strategy to others also requires students to think flexibly.

Inhibitory control involves sustained persistence in solving problems; resistance to distractor interference; being able to step back from a problem, abstract key information and develop a plan; and pausing to reflect on prior experiences to identify similar problems and strategies that might be used. Non-routine problem solving often requires learners to abstract essential text, values, and variables and ignore other non-essential information. Inhibitory control can also come into play through participation in the classroom community by listening to, making sense of, and contributing to group and whole class discourse.

Working memory involves recalling information and strategies to extend the retention of information. Strategies that support working memory include repetition of tasks, chunking smaller bits of information, and documenting one's thought process (e.g., showing your work). Given the focus of mathematics education on procedural fluency, working memory in mathematics curricula and instruction often (over)emphasizes activities such as completing practice sets and timed tests requiring rapid recall of symbols, representations, and procedures.

CONTEXT, APPROACH AND METHOD: INTENTIONS VERSUS REALITY

During the summer of 2021, six mathematics teachers committed to engaging in the co-design and eventual piloting of Spark Math. Three school administrators also participated in articulating the goals, norms, and outcomes for the co-design process. All the teachers involved worked in the same school in the midwestern United States and taught sixth grade (i.e., 11- to 12-year-olds). All the students enrolled in the school qualified for government support for breakfast and lunch (i.e., low income).

The data summarized in this paper includes digital artifacts developed during co-design meetings over the 2021-22 school year, including the development of a Community Charter, which will be described below. The goals and vision established in this digital document informed the co-design of lessons, activities and instructional sequences for sixth grade units focused on algebraic reasoning, ratios and proportional reasoning, and statistics. As co-designers, teachers also worked virtually with members of the project team on the creative ideation of lessons and activities, which provided some degree of trust, rapport, and mutual concern during the worst months of the pandemic, and later in the piloting of units and observation of teachers' use classrooms.

Teachers piloted the algebra and statistics units in January and May 2022, and piloted the ratio unit in August/September 2022. Decisions about content goals for each unit were made after reviewing state content standards; teachers also identified high-priority content goals for state testing. Teacher interviews were conducted over Zoom soon after they implemented Spark Math units. Classroom observations and professional development sessions were used to support the co-design process from curriculum design to implementation.

FINDINGS AND LESSONS LEARNED

Our original commitment to engage teachers as partners in co-designing mathematics curriculum provided the project team early indications regarding what teachers understood about EF processes, teachers' conceptions of less familiar mathematical representations (e.g., double number line, hangar model for representing and solving equations), and how they regarded proposed classroom activities such as games, group work, and projects.

In the sections below we describe select findings and lessons learned with respect to co-designing mathematics curriculum. These findings for co-design work include the importance of developing a Community Charter, ways in which explicit attention to executive functions processes can be integrated into mathematics education, the influence of the COVID-19 pandemic on the co-design and implementation of Spark Math, and the ways we managed the intensification of teacher work.

Developing a Community Charter

Early phase discussions between the project team, school administrators, and teachers included the joint development of a Community Charter (CC) in which collective interpretations of success, core values, equity, community norms, and norms for co-design participation were articulated. The CC was developed through multiple rounds of virtual discussions among four middle grades mathematics teachers, four school administrators, and eight researchers and curriculum designers with research backgrounds in mathematics education and/or education psychology. Whole group and breakout group discussions for each of the CC topics provided opportunities for individuals and groups to summarize goals and norms on a digital Jamboard. Ideas written on digital notes posted to the Jamboard were later synthesized into summary statements that were drafted, reviewed, edited, and confirmed by the participants.

The final purpose statement – i.e., what the group hoped to accomplish through this co-design process – included statements that centered on students' identity, agency, and interests, in addition to developing student recognition of EF processes:

> We endeavor to create mathematics education experiences that help students develop robust mathematical identities. We want students to see WHY they are learning math and to help them recognize the purpose of what they are learning. This can be accomplished by building math experiences that are inclusive and representative of students' lived experiences, as well as fostering a self-awareness of the executive function skills and habits of mind necessary to make connections between problem contexts and mathematical abstractions, and engage in authentic problem solving.

Particular attention was given to designing mathematical activities that could exemplify to students why we learn mathematics, using realistic problem contexts (van den Heuvel-Panhuizen & Drijvers, 2020; Webb & Peck, 2020) that were informed by students' interests. There was also a nearly unanimous understanding among co-designers regarding problem contexts and authentic problem solving as opportunities to develop EF skills.

Statements included in the CC also reflected early phase articulation and negotiation of how the co-designers identified success. Even though state testing results for schools often drive school and public perceptions of success, the community charter included several other noteworthy indicators of success.

> Success for students is achieved through greater interest (understanding the relevance and utilization of mathematics) and engagement (indicated by student joy, cultural affirmation, and identifying as a math person) in mathematics.
>
> After experiencing Spark Math:
>
> - more students will report that they believe math is for them;
> - more students will feel culturally affirmed in the math classroom;
> - more students will report a feeling of belonging in the math classroom;
> - students will demonstrate measurable academic growth;

- students will demonstrate increased ownership and excitement for math learning, increased engagement and perseverance in the classroom, and increased growth mindsets;
- students will be able to articulate and describe specific EF skills (working memory, inhibitory control, etc.) and know when they are using them in the classroom.

As expected, the criteria for success included “measurable academic growth,” which was interpreted as improvement in mathematics scores on state standardized exams, as well as interim benchmark measures. Even though this was deemed a high-priority goal, given our discussions with school administrators and teachers, one can see that this was not the only hoped-for outcome included under the criteria for success. Co-designers affirmed that students should be able to describe and recognize EF skills and argued that students should have a sense of belonging, and increased interest, ownership, and engagement in mathematics.

How executive function processes informed curricular decisions

Prior to the development of the CC, education psychologists with a background in executive function research provided an overview of EF concepts and processes to the rest of the co-designers from both neuropsychological and educational perspectives, then described how research on EFs included categorization of hot and cool EFs (Salehinejad et al. 2021). Cool EFs are processes that one might expect to utilize when engaged in non-provocative routine/traditional academic activities, such as solving a set of multi-step problems. Cool EF processes such as working memory, cognitive flexibility, ignoring irrelevant information, and inhibitory control might be utilized depending on the level of challenge of the set of tasks in relation to the learner(s). Hot EFs were described as more emotionally laden EF processes, such as impulse control, managing emotions, and delayed gratification.

Activities exemplifying ways in which we regularly used cool EFs included digit span tasks (where observers would be asked to recall numbers in order and in reverse), and the Stroop Task (which requires observers to recite the color of words, where the letters used to spell various colors does not match the actual color of the letters). Examples of games that leverage cognitive flexibility, working memory and inhibitory control were also shared (e.g., Set, Uno). Teachers soon recognized how they used EF processes extensively when teaching, and the group began to identify other examples in mathematics education that were affirmed by EF researchers on the team.

These experiences with EF exemplification influenced our discussions when developing the CC, and were instrumental in the decisions made regarding instructional activities in Spark Math units. For example, mathematical games were identified as a compelling opportunity for “sparking” student engagement and developing EF processes in a context that required students to make sense of various mathematical representations (Karanevich & Webb, 2024). A unique mathematics game was designed for each of the units that could be played by students in the classroom. Two games were also programmed for online play. Teachers interviewed by Karanevich and Webb (2024) described how these games increased student engagement with both EFs and mathematics content.

The influence of the pandemic as a co-design context

The global pandemic had a significant impact on the co-design, pilot, and revision process of the curriculum. Virtual and hybrid learning environments created a demand for the originally tactile materials to become digitized (Webb, 2022). The team leveraged other digital platforms, like an easy to navigate website, Google Slide decks, and PhET mathematics simulations (phet.colorado.edu) to support teacher implementation of lessons. Skye, a digital world to support students' self-guided learning and practice using videos, games, and other activities, was also co-designed and used by teachers and students. This digitizing approach offered more flexibility to teachers and students to access and interact with materials regardless of the modality of the learning environment.

The pandemic was also a significant barrier to onsite observation of teacher and student use of the curriculum through June 2022. Due to district health-related restrictions, in person observations of students and teachers did not begin until the second half of 2022. Due to these constraints, student and teacher feedback regarding student engagement, peer interaction, and use of games and simulations was gathered through online surveys, project team office hours (attended voluntarily by teachers), and post-implementation teacher interviews.

Intensification of teacher work

During the co-design process, neither the project team nor the teachers foresaw how the intensification of teacher work during the COVID-19 pandemic might influence the enactment of our EF-integrated mathematics curriculum. Successive rescheduling of when teachers would pilot the initial algebra unit was blamed on other school-related factors. However, during the pilot of the first unit in January 2022, it was evident that multiple factors that intensified teacher work had now come into play. These factors included teachers' increased attention to students' socioemotional well-being, the school's heightened attention toward student learning loss and recovery of mathematics skills that were to be covered on state tests, and limitations in teachers' capacity to prepare and plan mathematics lessons that involved greater attention to students' ideas and informal and pre-formal representations.

Different priorities emerged during the pandemic that forced the project team to manage expectations regarding teacher implementation of units. Given that most of the co-designers were teaching in K-12 schools or universities, there was a shared understanding of the realities of teaching, which resulted in renegotiating when to pilot and how to pilot.

A curriculum adaptation and shift in expectation caused by the pandemic was the teachers being unable to implement an entire unit, which included contextual storylines that connected lesson sequences. Through conversations with co-design teachers, we agreed to revise all the original units (15 – 20 lessons each) into clusters of independent modules (about 3-4 lessons each), in which each module addressed the content goals for the unit. That is, for a five-module unit, teachers could decide to teach only modules B, D, and E. Units were adapted to reduce, as much as possible, any interdependency between the mathematical content and contextual storylines of each module. This adaptation recognized the challenges teachers faced in piloting new instructional resources and provided teachers with greater agency in deciding how to address content goals with their students.

DISCUSSION

Despite the additional time required to involve teachers and students in the co-design of Spark Math, engaging in co-design as early as possible in the development of the curriculum allowed the project team to juxtapose the intentions of mathematics educators and EF researchers with the perspectives, interests and practical realities of teachers and students. The unintended benefit of the pandemic was a necessary delay in the pilot phase. This longer co-design ideation phase provided the time needed for creative integration and conceptualization of instructional sequences.

Given the turmoil and intensification of teacher work during the pandemic, it might appear unreasonable to recommend the integration of EF processes into mathematics curriculum as another goal for mathematics educators to consider. One challenge, therefore, is finding ways to infuse core EFs in mathematics education that exemplify how instructional resources and pedagogical approaches might accomplish sufficient integration of EFs in ways that are beneficial, meaningful, and interesting to teachers and students. One silver lining of the experience of the global pandemic in education was the recognition of the need to attend to students' socio-emotional learning in ways that may have been underappreciated by the greater mathematics education community. A benefit of this increased attention to student well-being has been teachers' use of classroom discourse that includes norms for supporting each other, attending to students' interests, and discussing ways to manage academic workload. Similarly, the discussion of EFs should not be presented as something new to learn or yet another set of academic terms to memorize. Rather, EFs should be recognized as student assets that are worth sharing and discussing. The ways in which students describe how they support working memory, leverage their cognitive flexibility, and exhibit inhibitory control should be in students' terms (and then the teacher can decide if or when it is appropriate to relate to formal language). Cross-disciplinary teams in other countries interested in integrating EF and mathematics education could share design principles and exemplars of resources to catalyze this work.

As the U.S. school system moves further away from the uncertainties caused by the pandemic, mathematics teachers are still coping with the intensification of work caused by demands from administrators, parents, and the needs of their students. To lessen demands on teachers, we redesigned units to offer greater flexibility in how teachers used the curriculum. Instead of a replacement unit approach requiring teachers to implement nearly all unit lessons from start to finish, we found that the modularized approach supported teacher agency in how they chose to address content goals and their perceptions of students' needs and interests. Research on teacher use of innovative curricula may want to consider ways to recognize the role of teacher agency in their use of instructional materials.

Lastly, even though teachers may not typically see themselves as "professional designers," involving teachers in curriculum planning, lesson design, and iterative revision cycles is a professional learning opportunity for teachers that can lead to a deeper understanding of the relationship between curriculum, teaching, and learning. It is also worth noting that teachers asked students to provide input regarding problem contexts, games, and digital activities throughout the development of the curriculum. When the research team visited schools, we continued to ask students for advice regarding ways to adapt activities and suggest other project contexts. These connections made with designers enabled some students to see themselves as contributors to Spark Math. However, teachers also observed how asking students about their interests could inform ways to design and adapt activities and select instructional sequences. Co-design experiences such as these can lead teachers to see themselves as designers, or at the very least appreciate the decisions that come into play when designing mathematics curricula.

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