Relations Between Math Vocabulary and Error Patterns When Solving Math Word Problems

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Background

- Math vocabulary is foundational to understanding of math concepts[1,2,3] and has been positively associated with students' performance on equations and word problems.[4]
- Instruction on math vocabulary predicts improvements in students' math problem-solving performance.[2,3]
- When completing word problems, children's errors provide insight into their understanding of problem-solving processes.[5]
  - Errors in computation indicate a conceptual understanding but need for additional practice or training in the calculation procedure.[5]
  - Other error types may indicate conceptual misunderstanding.[5]
- Learning more about the relations between math vocabulary and math performance may inform future research regarding targeted intervention approaches.

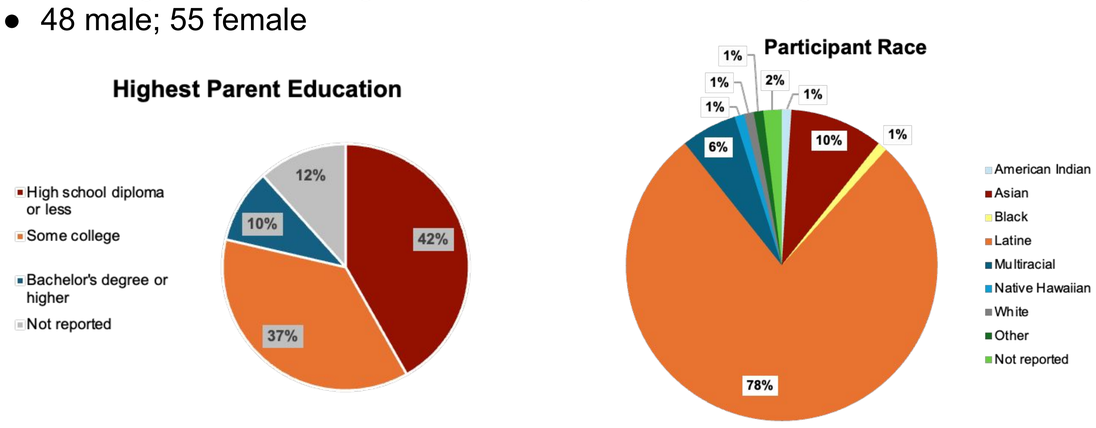
The Present Study

This study examined the extent to which math vocabulary is related to patterns of math word problem solving across 3rd-7th grades and, specifically, whether math vocabulary correlates with error pattern types across problem schemas.

Method

Participants:

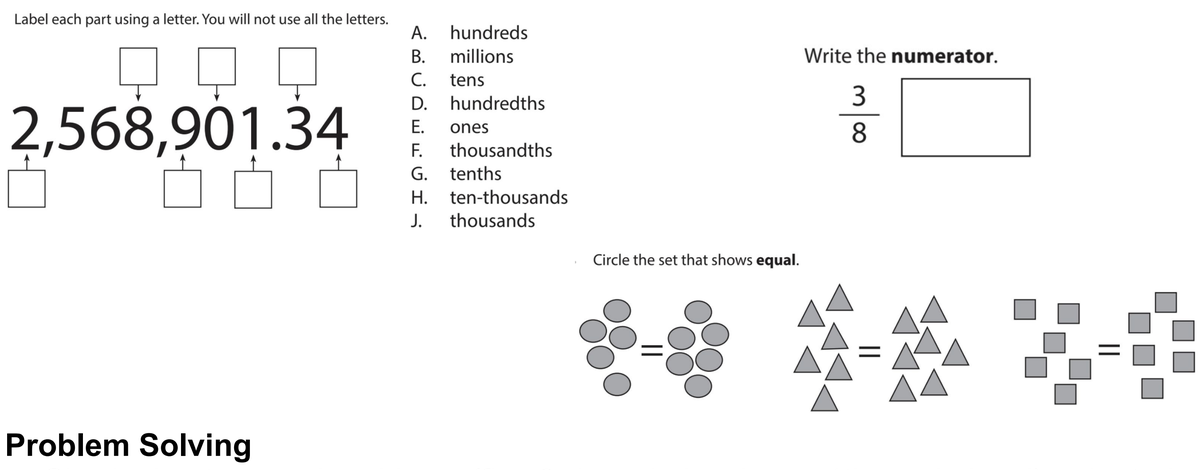
- 103 students from seven classrooms at one school in California
- 16 3rd graders; 35 4th graders; 32 5th graders; 20 7th graders
- 48 male; 55 female



Researchers administered the measures below in students' classrooms in either November 2022 or February 2023 as pretests for a pilot study of the "Our Mathematical World" program.

Math Vocabulary

- 27 K-4th grade-level math vocabulary terms (adapted from Powell et al., 2017)[1]
- Example items:



Problem Solving

- Six single-step word problems (five 3rd-grade level, one 4th-grade level), adapted from Powell et al. (2020),[4] coded for error type.[5]
- Schemas of total (*N* = 2), change/difference (*N* = 2), and equal groups (*N* = 2):
  - Total: The animal park has 68 tigers and some giraffes. If the total number of tigers and giraffes is 99, how many giraffes are there?
  - Change/Difference: Kavon rode the roller coaster 44 times. His brother rode the roller coaster 25 times. How many fewer times did Kavon's brother ride the roller coaster?
  - Equal Groups: Mr. Lopez sold 5 boxes of peaches at the farmer's market. Each box holds 12 peaches. How many peaches did Mr. Lopez sell?

Results

Associations Between Math Vocabulary Accuracy and Problem-Solving Error Type Within Each Schema

		Sound Conceptual Errors	Poor Conceptual Errors
	n	r_p	r_p
Total	28	.58***	-.38*
Change/Difference	26	.52**	-.25
Equal Groups	28	.11	.03

Note. Controlling for Grade. $^*p < .05$; $^{**}p < .01$; $^{***}p < .001$

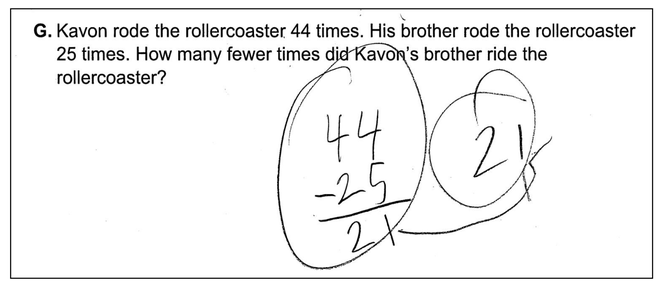
Error types in incorrect responses by schema:

- Math vocabulary accuracy positively predicted *sound* conceptual errors (e.g., computation errors) for:
  - "Total" (r_p = .58, p = .001)
  - "Change/Difference" (r_p = .52, p = .007)
- Math vocabulary negatively predicted *poor* conceptual errors (e.g., omission errors) for "total" (r_p = -.38, p = .048) but no other schemas.

- For the full sample, math vocabulary accuracy (Min. = .04, Max. = .96, *M* = .41, *SD* = .27) predicted problem-solving accuracy (Min. = 0.00, Max. = 1.00, *M* = .59, *SD* = .34) across schemas, even when controlling for grade (r_p = .63, p < .001).

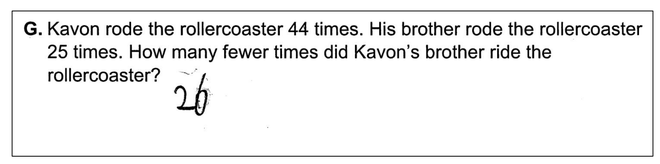
Sound Conceptual Errors:

- Attempted correct operation and calculated incorrectly
- Demonstrated understanding of the problem-solving process (sets up their work and begins to solve), but does not finish (e.g., running out of time)
- Transcribed a number from the problem incorrectly and computed correctly from there



Poor Conceptual Errors:

- Copied a number from the problem
- Skipped/did not attempt to solve
- Wrote an answer with no work where the process cannot be inferred



Discussion

- The strong positive association between math vocabulary and overall problem-solving accuracy is consistent with prior work.[4]
- Within incorrect responses, the positive association between math vocabulary and sound conceptual errors for total and change/difference problems suggests a stronger relation between math vocabulary skills and these simpler schemas (requiring addition and subtraction) compared to more advanced schemas such as equal groups (requiring multiplication).
- Total and change/difference schemas are introduced earlier in children's academic careers, which is consistent with prior work indicating that the association between math vocabulary and computation is stronger in third grade than in fifth.[1]

Limitations and Future Directions

- This study only examined single-step problems at one time point and examined overall math vocabulary accuracy. Analyses did not account for the nuances that exist within math vocabulary, such as technical and subtechnical terms.[6]
- Further research should examine the relation to multi-step problem solving, as well as students' response to interventions, which can inform the understanding of math vocabulary as related to student learning.
- Further research should also explore associations between problem-solving performance and subcategories of math vocabulary.

Acknowledgements

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