Educator Guide to Self-interpretive Feedback

Introduction

This document introduces self-interpretive feedback as a response technique for educators to use when students present an answer in mathematics that may contain errors. Rather than immediately correcting or providing hints, self-interpretive feedback involves the educator articulating their interpretation of the student's response and asking the student to confirm their meaning. This method is proposed as an entry point to richer, more constructive conversations with students. The following sections will provide examples of self-interpretive feedback, describe its various advantages—including preserving mathematical rigor and reducing student anxiety—and offer additional justification and guidance for its implementation in the classroom and in educational software.

Self-interpretive feedback

Consider the following student statement:

"⅔ of the apples are red."

There are many good ways an educator could respond, such as "Not quite - why don't you look at the problem we did before?" Educators absolutely should use this and other kinds of feedback. However, based on our experiments, we suggest educators consider first responding with **self-interpretive feedback**:

"I read your answer as saying there are 2 red apples out of 3 total apples. Is that what you meant?"

Other examples of self-interpretive feedback include:

- [Educator highlights the circumference of a circle] "I read your answer as saying this part of the circle is 2 meters long, if the radius is 1 meter long. Is that what you meant?"

- “I read your answer as saying that 5 is the only number I could square to get 25. Is that what you meant?”
- [In response to the answer “0.5%”]: “I read your answer as saying that if I flip a coin 100 times, I would see heads 0.5 times and tails 99.5 times. Is that what you meant?”

Advantages of self-interpretive feedback

- Preserves the mathematical rigor of the problem, unlike giving hints or partial answers
- Gives students a chance to correct their answer before final submission to the educator
- Invites a mutual conversation, unlike “Try again”
- Very easy to program in computer software compared to more complicated kinds of responsive feedback. In our experiments, also leads to the best gains on a difficult follow-up fraction modeling activity compared to more standard software feedback
- Clarifies and reinforces the meaning of important mathematical vocabulary - particularly relevant for English language learners
- Models good communication and conflict resolution behavior
- Less likely to be interpreted as “You’re bad at math,” one of the worst types of feedback

Additional justification and framing

We hope these examples and advantages convince you to increase your use of self-interpretive feedback, especially as a gateway to having richer conversations with your students about what they are thinking and what they are trying to communicate. If you would like to read more about why we tried this kind of feedback in the first place and see additional justification for our specific phrasing, feel free to read on.

The importance of feedback

When a student proposes a (possibly incorrect) answer to a math question, there are many ways an educator could respond. Different responses can have drastically different effects on students which compound over time, some of which may not be intuitive. One of the marks of an expert educator is thus the ability to respond well, in real time, to widely varying student work.

For example, here are some feedback types studied in literature:

Type	Example	Effect
Positive comparative	“You’re in the top 5% of surgery residents in our program.”	Increased intention to become a surgeon.
Cognitive	“Your program has a bug on line 45.”	Increased short term learning gains.
Mastery (“growth”)	“I see you worked really hard on this	Increased motivation to

mindset	drawing."	practice, increased long term learning gains.
Positive emotional	"You're doing so great! You're very smart at math."	**Decreased** learning gains and motivation to practice (unless students have low self-efficacy)
Stereotyping	"Men don't need to learn to cook"	Decreased learning gains, decreased motivation to practice, increased stereotype threat

Math: a language for numerical arguments

Math is a language designed to communicate certain kinds of arguments in a precise way. Solving an equation, for example, is making an argument that some values of x make the original equation true. Showing one's work is justifying the logical steps that lead to the conclusion that x must be one of a set of specific values.

Imagine you were asked to argue for free school lunches in a language you didn't know, i.e. Norwegian. Your argument could be flawed *linguistically* - some kind of grammar error or misuse of vocabulary - or it could be flawed *logically*, in which case your entire argument is unsound. Linguistic flaws make it harder to understand the argument, but the argument could still be a good one. Logical flaws mean the argument has to be discarded entirely.

The same problems happen in math all the time, which is not surprising because math is a language designed to make arguments. However, because math is so concise, it is very easy to mistake a linguistic flaw for a logical one. As such, we often jump to the assumption that students who make errors are making mathematical errors - fundamental mistakes in logic that mean they are not thinking correctly. However, a wrong answer could also be due to incorrect usage of the *language* of math, even while students have a clear and correct understanding of the *logic* of math.

Why interpretive feedback?

Interpretive feedback is meant to help educators distinguish between linguistic and logical errors. In the apple feedback example, a student might say, "What? No, there are 2 red apples and 3 green apples." In this case, the student is logically comprehending the situation and simply misunderstands how fractions are interpreted in this context: as parts of a whole, rather than comparing parts to parts. The educator could then respond, "OK, so if I always read the fraction 2/X as meaning 2 of the apples are red out of X total apples, what would you put in the denominator?" They could then have a discussion with the student and test different denominators until settling on ⅖ as leading to a correct interpretation.

Using “I” statements

There are many ways to phrase interpretive feedback. In the apple example, the educator could have said “Your answer states that 2 out of 3 apples are red. Do you think that’s reasonable?” While we personally have not run experiments on different phrasing of interpretive feedback, there is research that “you”-type phrasing can be worse for learning. For example, beginner software programmers learn better when compiler errors are phrased as the *computer* being unable to understand what the programmer was trying to do, rather than the typical phrasing that the programmer wrote their program incorrectly.

This actually might not be that surprising, given the usual guidance for conflict resolution: use “I” statements and avoid statements that could be interpreted as accusations, especially accusations impugning the other person’s character. The “You” statement above is very close to claiming that the student believes there are only 3 apples, 2 of which are red, and that their thought process is unreasonable. Taken the wrong way and reinforced over years, the student might learn that they are Bad At Math and shy away from situations where their Badness could be exposed. In a way, it is the difference between saying “Oh cool, we have to watch Star Wars at my house!” compared to “You haven’t heard of *Luke Skywalker?* Do you live under a rock?”