Proceedings for the 51st Annual Meeting of the Research Council on Mathematics Learning

Celebrating 51 years of Research on Mathematics Learning

February 29 – March 2
Columbia, South Carolina

RCML Officers & Conference Team

PRESIDENT, 2023-2025
Sarah Pratt
Baylor University
Waco, TX
Sarah_Pratt@baylor.edu

PAST PRESIDENT, 2023-2024
Travis Olson
University of Nevada at Las Vegas
Las Vegas, NV
travis.olson@unlv.edu

VICE PRESIDENT FOR CONFERENCES, 2022-2024
Jamaal Young
Texas A&M University
College Station, TX
jamaal.young@tamu.edu

VICE PRESIDENT FOR PUBLICATIONS, 2021-2024
Colleen Eddy
University of North Texas
Denton, TX
colleen.eddy@unt.edu

VICE PRESIDENT FOR PUBLICATIONS – ELECT, 2023-2024
Babette M. Benken
Cal State University Long Beach
Long Beach, CA
babette.benken@csulb.edu

TREASURER, 2022-2026
Tyrette Carter
North Carolina A&T University
Greensboro, NC
tscarte1@ncat.edu

SECRETARY, 2023-2025
Tina Mitchell
Delaware State University
Dover, DE
tmitchell@desu.edu

HISTORIAN
William Speer
University of Nevada at Las Vegas
Las Vegas, NV
william.speer@unlv.edu

INVESTIGATIONS EDITOR
Jonathan Bostic
Bowling Green State University
Bowling Green, OH
jbostic@bgsu.edu

INVESTIGATIONS ASSOCIATE EDITORS
Drew Polly
University of North Carolina at Charlotte
Charlotte, NC
abpolly@uncc.edu

Jennifer Cribbs
Oklahoma State University
Stillwater, OK
jennifer.cribbs@okstate.edu

NEWSLETTER CO-EDITORS
Tina Mitchell (2021-Present)
Delaware State University
Dover, DE
tmitchell@desu.edu

Byungeun Pak (2023-Present)
Utah Tech University
St. George, UT
b.pak@utahtech.edu

PROCEEDINGS EDITOR
Dennis Kombe
Cal State University, Monterey Bay
Seaside, CA
dkombe@csumb.edu

PROCEEDINGS CO-EDITOR
Ann Wheeler
Texas Woman's University
Denton, TX
AWheeler2@twu.edu

CONFERENCE COMMITTEE
Seanyelle Yagi (2021-2024)
University of Hawaii at Manoa
Honolulu, HI
slyagi@hawaii.edu

Barbara Allen-Lyall (2021-2024)
Manhattanville College
Purchase, NY
barbara.allenlyall@mville.edu

Nickolaus Ortiz (2022-2025)
Georgia State University
Atlanta, GA
nortiz1@gsu.edu

Karen Zwanch (2022-2025)
Oklahoma State University
Stillwater, OK
karen.zwanch@okstate.edu

Lucas Foster (2023-2026)
Northeastern State University
Broken Arrow, OK
fosterlb@nsuok.edu

Jennifer Gonzales (2023-2026)
Northside Independent School District
San Antonio TX
Jenny.Gonzalez@nisd.net

2024 Local Host Co-Chairs
Sean Yee
University of South Carolina
Columbia, SC
yee@math.sc.edu

Christie Martin
University of South Carolina
Columbia, SC
martinc1@mailbox.sc.edu

THANK YOU TO OUR REVIEWERS

Babette Benken	Tonya Garrett	Alesia Moldavan
Ken Butler	Nico Gòmez Marchant	Ayse Ozturk
Georgia Cobbs	Andrew Goodman	Byungeun Pak
Angie Crawford	Pat Jordan	Alyssa Sayavedra
Jen Crooks Monastra	Puskar Joshi	Dawn Teuscher
Colleen Eddy	Pavneet Kaur Bharaj	Carrie Toreky
Kristian Edosomwan	Karl Kosko	Sandra Trowell
Brian Evans	Karl Kruczek	Patty Wagner
Timothy Folger	Melinda Lanius	Ann Wheeler
Lucas Foster	Hope Marchionda	Nicholas Wong
Ryan Fox	Tina Mitchell	Sean Yee
		Alan Zollman

For citation of articles in the Proceedings:

Authors. (2024). Title of paper. In D. Kombe, & A. Wheeler (Eds.), *Proceedings of the 51st Annual Meeting of the Research Council on Mathematics Learning.* (pp.##-##). RCML.

Editorial Assistance:

Pavneet Kaur Bharaj, California State University, Bakersfield, CA.

Graduate Student Editorial Assistant:

Jennifer T. H. Hylemon, University of North Texas, Denton, TX

Publication Acceptance Rate:

Accepted 14 manuscripts out of 26 submissions. Acceptance Rate of 54% (last year 48%)

Please Note:

RCML History

The Research Council on Mathematics Learning, formerly The Research Council for Diagnostic and Prescriptive Mathematics, grew from a seed planted at a 1974 national conference held at Kent State University. A need for an informational sharing structure in diagnostic, prescriptive, and remedial mathematics was identified by James W. Heddens. A group of invited professional educators convened to explore, discuss, and exchange ideas especially in regard to pupils having difficulty in learning mathematics. It was noted that there was considerable fragmentation and repetition of effort in research on learning deficiencies at all levels of student mathematical development. The discussions centered on how individuals could pool their talents, resources, and research efforts to help develop a body of knowledge. The intent was for teams of researchers to work together in collaborative research focused on solving student difficulties encountered in learning mathematics.

Specific areas identified were:

1. Synthesize innovative approaches.
2. Create insightful diagnostic instruments.
3. Create diagnostic techniques.
4. Develop new and interesting materials.
5. Examine research reporting strategies.

As a professional organization, the **Research Council on Mathematics Learning (RCML)** may be thought of as a vehicle to be used by its membership to accomplish specific goals. There is an opportunity for everyone to actively participate in **RCML**. Indeed, such participation is mandatory if **RCML** is to continue to provide a forum for exploration, examination, and professional growth for mathematics educators at all levels.

The Founding Members of the Council are those individuals that presented papers at one of the first three National Remedial Mathematics Conferences held at Kent State University in 1974, 1975, and 1976.

Table of Contents

IMPROVEMENT IN MATH PROBLEM SOLVING IS MODERATED BY WORKING MEMORY

Rick Bryck
Landmark College
rickbryck@landmark.edu

Sam Rhodes
Georgia Southern University
srhodes@georgiasouthern.edu

The purpose of this study was to investigate whether middle school students' working memory capacity influenced the impact of an intervention aimed at improving their problem solving proficiency. The sample included a total of 179 grade 6 and 7 students from a middle school located on the West Coast. The results suggest that gains from the problem solving intervention were moderated by working memory capacity, with students with higher initial working memory capacity showing the largest gains on problem solving proficiency.

Background

Gaining proficiency in problem solving (PS) is a key outcome in K-12 mathematics (NCTM, 2014). Proficiency in PS—operationalized herein as the ability to successfully solve cognitively demanding mathematics word problems—has been shown to be related to numerous factors such as metacognition, executive function (EF), content knowledge, strategic thinking, and affective characteristics such as student beliefs (Chapman, 2015; Rhodes et al., 2023; Schoenfeld, 2013). Specifically, PS requires that students decode tasks, transpose problem information using mental models, process information, and implement plans (Singer & Voica, 2013), all of which involve cognitive (particularly EF) processes. However, despite increasing evidence of the various cognitive and affective factors that influence PS, little remains known about how best to improve PS performance in students (Lester & Cai, 2016).

Working memory (WM), considered a keystone EF ability (Friedman & Miyake, 2017), has been implicated as a critical factor in mathematics (e.g., Bull & Lee, 2014; Raghubar et al., 2010). WM is a mental workspace used to maintain short-term focus of attention and manipulate this information, often in the service of accomplishing complex cognitive processing (Baddeley & Hitch, 1974). A relationship between WM ability and academic achievement comes from both theoretical accounts (e.g., Miyake & Shah, 1999), as well as empirical studies demonstrating correlations between mathematic performance and WM ability, including meta-analyses (e.g., Friso-van den Bos et al., 2013) and longitudinal studies (Alloway & Alloway, 2010). EF skills can depend on contextual factors, and with appropriate training and scaffolding can be developed and strengthened.

Naturally then, it has been posited that strengthening WM might in turn improve math performance. However, much is still unknown about the precise contribution of WM to math, and in particular to mathematical PS proficiency. This includes whether, and how, variations in WM may affect targeted math interventions. Understanding these connections can inform future math pedagogy, such as whether differentiated WM support is needed. Thus, the purpose of the present study was to investigate the role of WM capacity as moderator of gains in mathematical PS, in the context of a larger PS intervention study. Guiding this work, we posed the following research question: Does an individual's baseline working memory capacity affect their gains in mathematical problem solving performance?

Theoretical Framework

Working memory is a critical factor in many theories of information processing and cognition (e.g., Anderson et al., 1997; Meyer & Kieras, 1997). It has been linked to the ability to focus attention, in the face of distractors, to important information and details relevant to one's current goal, especially in the formation of new concepts and how multiple concepts relate–such as is required for mathematics (or any) learning (Cowan, 2014). This idea is further supported by research demonstrating a relationship between individual variation in working memory capacity (the number of items one can retain) and mathematics ability (e.g., Friso-van den Bos et al., 2013; Raghubar et al., 2010). Additionally, evidence suggests that developing EF skills, including WM, support the development of math learning and problem-solving, and vice versa (Clements et al., 2016; Zelazo et al., 2017). Recent evidence also supports the *combined* importance of metacognitive ability and EF (along with student beliefs and prior content knowledge) in the support of proficient PS (Rhodes et al., 2023). Yet little is known how individual differences in WM ability may impact math PS proficiency.

Description of Intervention

The present study was part of a larger study which aimed to improve mathematical PS performance in middle school students. In this larger study, students in an intervention group used a PS application that scaffolded and targeted EFs and metacognition within a four-phase attack strategy that was based on the work of Pólya (1945/2014). Scaffolds and supports included breaking the problem down into the four phases noted above, asking students what they notice and wonder about the problem, prompting students to explicitly consider what the problem was asking them to do, having students journal their plans for solving the problem while providing them with sample sentence stems, and having students explain and record their solution. The results come from a larger Pre vs. Post assessment (separated by about five months) study which showed that the intervention group significantly improved on mathematical PS performance when compared to students in a business-as-usual control group, as measured by the PS measure described below (see Rhodes et al., in preparation). Thus, the purpose of the present study was to expand this work by exploring whether the gains seen in the intervention group were moderated by working memory.

Methodology

Participants

The participants in the study were 6th and 7th grade students from a single school, referred to herein as Beach View. Beach View Middle School is located in a large, suburban school district from a West Coast State. All mathematics teachers at the school were offered the chance to participate in the study, along with all students enrolled in classes taught by participating teachers. Of these students, 92 6th grade students and 87 7th grade students completed both measures and are included in the analyses reported herein. The students self-identified as girl (n = 103), boy (n = 66), non-binary or prefer to self-identify (n = 5), prefer not to say (n = 3). Students self-identified (note multiple categories could be selected) as African-American or Black (n = 19), Hispanic, LatinX, or, Mexican (n = 131), Asian (n = 19), Chaldean or Middle Eastern (n = 69), Native American or Alaska Native (n = 5), Pacific Islander (n = 3), White (Non-Hispanic; n = 17), self-identified (n = 34), or prefer not to say (n = 33).

Measures and Scoring

Executive Function. The Adaptive Cognitive Evaluation (ACE;) was used to measure students' EFs; evidence supporting ACE as a valid measure of EF is presented in existing literature (Younger et al., 2022). The ACE is comprised of gamified, computer-based versions of well-known tasks that measure core EFs such as working memory, cognitive flexibility, and inhibitory control. Within the present study, working memory was the variable of interest and was measured using a change detection task (Luck & Vogel, 1997), with the key measure of interest being "K," an estimate of one's visual-spatial working memory capacity (i.e., the number of visuo-spatial items one can hold in mind at one time). K was calculated using the standard formula computing hits minus false alarms in the set size 2 condition, thus scores can range from

0 to 2. The set size 2 condition was used given average performance in the set size 4 condition fell below chance levels of performance.

Problem Solving Measure. Problem solving was measured using a 3-item test that consisted of problems that were written by Illustrative Mathematics (IM) and that was administered outside of the application used as part of the intervention. The items were chosen based on three criteria. Specifically, the problems 1) had a high degree of cognitive demand as assessed by the Smith and Stein (2018) framework; 2) align to priority standards within the district's pacing guides; and 3) offer opportunities for students to show or explain their process for solving the problems. If a selected problem did not offer sufficient opportunities to illuminate students' thinking, slight modifications were made to the directions of those problems. Given that the intervention was aimed at supporting students in learning the process of problem solving rather than any specific content or type of problem, problems were not explicitly aligned to any aspect of the intervention outside of the three aforementioned criteria. Each students' work was scored two ways: accuracy (total correct solutions) and understanding (the level of correct relevant mathematical thinking that the student demonstrated, regardless of answer accuracy). Fleiss' kappas were calculated to measure interrater agreement on the understanding scoring: .961 and .703 for 6th grade and .880 and .842 for 7th grade, for accuracy and understanding, respectively. This alignment to the problem-solving framework and interrater agreement provides evidence of validity and reliability related to the PS measure used.

Data Analysis. To start, 1-tailed Pearson correlations were used to examine the relationships between WM baseline (Pre-Test) scores and student scores on the PS measure. Given individual differences in WM have been shown to correlate with academic performance, including general mathematic ability, we were interested in examining whether changes in problem solving were moderated by students' baseline working memory score, we first computed median K (WM capacity) scores for 6th grade and 7th grade independently (to account for presumed developmental differences across the grades). A median split procedure was used, for each grade, such that students were split into "below-median" and "above-median" WM groups, using their PRE K score. Separate two by two multivariate analyses of variances (MANOVA) were calculated for the IM Accuracy and IM Understanding dependent variables, using the factors of wave (Pre vs. Post assessment) and the between-subjects WM group factor (low vs. high).

Results

Pearson correlations were low and non-significant when comparing WM baseline (Pre-Test) to pre-test PS scores (n = 179) with r = -.019, p = .399 for accuracy, and r = .056, p = .229 for understanding. However, the correlations comparing WM baseline (Pre-Test) scores to PS scores on the post-test were stronger, and significant, with r = .157, p = .018 for accuracy, and r = .207, p = .003 for understanding. In addition, we found small and significant correlations between PRE K scores and improvement in IM Accuracy and Understanding (Post minus Pre scores, on each measure, r = .150, p = .045 and r = .188, p = .012, respectively. At the multivariate level, Mahalanobis Distance was used, and one outlier was noted and removed and the MANOVA was re-run. The final results are reported below.

Figures 1 and 2 show mean scores by wave (pre vs. post) and WM group for IM Accuracy and Understanding, respectively. Higher scores for POST compared to PRE ("intervention effect") were observed, with this difference being more pronounced in the group with higher PRE WM capacity (K) scores, in both the IM Accuracy and Understanding measures. The interactions for wave by WM group were significant, for both IM Accuracy $F(1,176) = 6.113$, $p = .014$, $\eta_p^2 = .034$ and IM Understanding $F(1,176) = 8.042$, $p = .005$, $\eta_p^2 = .044$.

Table 1.

Mean and Standard Deviation Scores for IM Accuracy and Understanding

	N	Pre: IM Accuracy	Post: IM Accuracy	Pre: IM Understanding	Post: IM Understanding
Below Median K Group	92	.674 (.64)	.842 (.61)	.348 (.34)	.553 (.41)
Above Median K Group	86	.657 (.67)	1.11 (.62)	.414 (.33)	.811 (.54)

Figure 1.

IM Accuracy Scores by WM group and Time (Pre vs. Post)

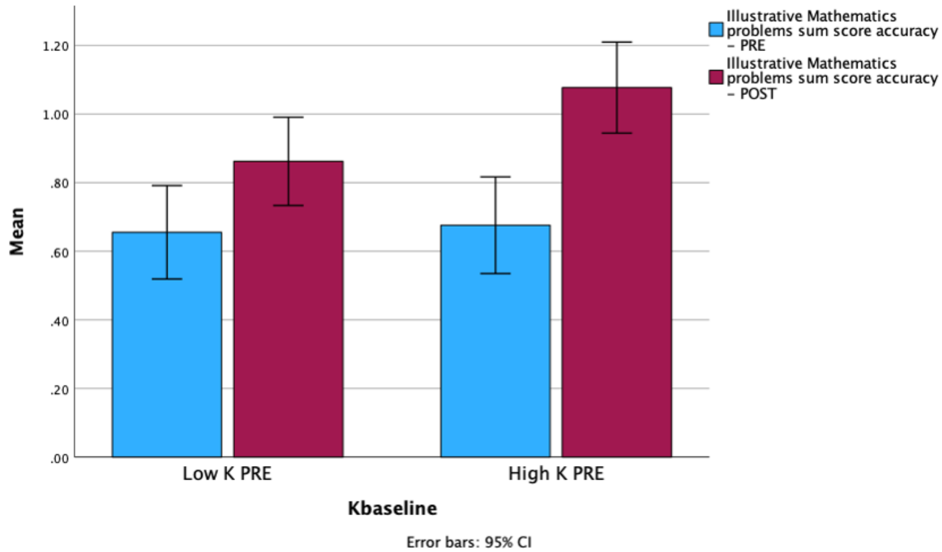
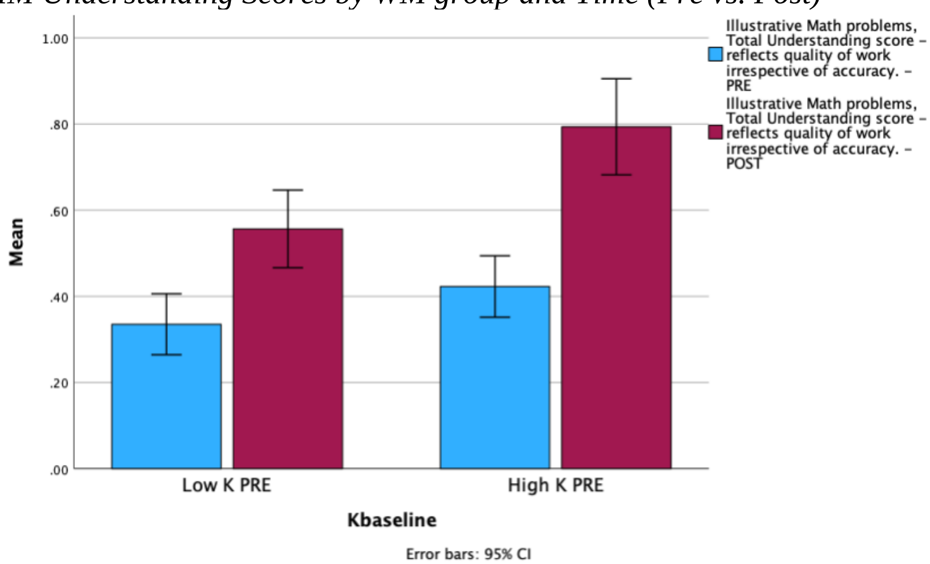


Figure 2.

IM Understanding Scores by WM group and Time (Pre vs. Post)



Discussion

The purpose of the present study was to explore whether students' initial WM capacity moderated the impact that a PS intervention (aimed at all students) had on students' PS performance. The results suggest that students' initial working memory capacity moderated the effectiveness of the PS intervention. Although students in both the below-median WM group and students in the above-median WM group significantly improved their PS performance across accuracy and understanding, that improvement was significantly higher for students in the above-median WM group. Significant correlations between WM scores and gains in PS strengthens this argument.

In seeking to interpret these results, it is important to note that baseline WM was not correlated to PS scores at pre-test, but was correlated to PS scores at post-test, with students in the above median WM baseline group showing significantly more growth. In addition, prior research suggests that problem solving involves numerous cognitive processes such as decoding data, creating mental models, and applying techniques to solve problems (Singer & Voica, 2013) – all of which are likely to put a high demand on working memory. Taken together, we theorize that students with higher WM capacity may have more effectively encoded, and then retrieved, the intervention scaffolds when needed from long-term memory. When the WM demands of doing so are taken into account, in conjunction with the cognitive demands inherent to problem solving, it stands to reason that students with a high WM capacity would be better equipped to retrieve and utilize the scaffolds when they were no longer being explicitly provided to them. Although students in the below-median WM group still improved their PS performance, the fact that they improved less than students in the above-median WM group may suggest that they were able to retain and utilize only a subset of the scaffolds, and/or were less effective in applying those scaffolds outside of the intervention itself.

Students with higher WM may also be better equipped to deal with the relatively high cognitive load (WM demands) inherent in the PS intervention platform. In other words, they were better able to process the multitude of information presented in each of the four-phases of the intervention program and thus better utilized the embedded scaffolds. This could also explain the results presented here, either alone, or in conjunction with the above explanation about transfer of the scaffolds to situations where they were not present.

These results have broad implications for classroom instruction related to mathematical PS in the middle grades. Specifically, they provide evidence that WM is critical to consider when designing PS interventions. The results may also suggest that teachers and researchers need to explicitly consider how to support students in retaining and transferring WM scaffolds beyond intervention conditions to ensure that they can apply these skills to other class activities and assessments. Teachers should be cognizant of varying levels of WM capacity in their students and provide EF scaffolds and supports in the moment. This will in fact inform future iterations of the broader intervention, such that additional supports will be provided to reduce cognitive load and WM demands during different phases of the program.

General approaches include awareness of extraneous load in math problems (e.g., overly complex, or unneeded wording), build in more time to process problems, and add prompts for taking action and/or reflection (e.g., "what is the next step I should take in solving this problem?"). In short, teachers should consider ways in which extraneous WM demands can be lessened without changing the rigor of problems. In conclusion, the results suggest that WM is an important variable to consider in for improving PS proficiency in middle school students.

Limitations and Avenues for Future Research

There are several limitations regarding the current study. First, students' WM was measured at a single point in time and was, therefore, treated as a trait-based variable. However, new research has suggested that EFs may be state-based. Thus, future studies should consider utilizing in-the-moment measure of EFs or measuring EFs at numerous points during studies rather than just at pre- and post-test. Secondly, the design of the present study limited the researchers' ability to explore causal effects and thus future studies may consider how to gain more nuanced understandings of these relationships through true experimental designs and/or the use of qualitative methods such as cognitive interviews. Other future studies could explore the efficacy of differentiating the types of EF scaffolds given during mathematics PS instruction based on individual student's WM ability.

Acknowledgements

The research reported here was supported by the EF+Math Program of the Advanced Education Research and Development Fund (AERDF) through funds provided to CueThinkEF+. The perspectives expressed are those of the authors and do not represent views of the EF+Math Program or AERDF.

References

Alloway, T. P., & Alloway, R. G. (2010). Investigating the predictive roles of working memory and IQ in academic attainment. Journal of Experimental Child Psychology, 106(1), 20–29. https://doi.org/10.1016/j.jecp.2009.11.003

Anderson, J. R., Matessa, M., & Lebiere, C. (1997). ACT-R: A theory of higher level cognition and its relation to visual attention. *Human-Computer Interaction*, *12*(4), 439–462. https://doi.org/10.1207/s15327051hci1204_5

Baddeley AD, Hitch G (1974). Bower GH (ed.). Working Memory. Psychology of Learning and Motivation. Vol. 2. Academic Press. pp. 47–89. doi:10.1016/S0079-7421(08)60452-1. ISBN 978-0-12-543308-2. OCLC 777285348.

Bull, R., & Lee, K. (2014). Executive Functioning and Mathematics Achievement. Child Development Perspectives, 8(1), 36–41. https://doi.org/10.1111/cdep.12059

Chapman, O. (2015). Mathematics teachers' knowledge for teaching problem solving. *Lumat.Fi, 3*(1), 19-36. Retrieved from http://lumat.fi/index.php/lumat-old/article/view/38
Clements, D. H., Sarama, J., & Germeroth, C. (2016). Learning executive function and early mathematics: Directions of causal relations. *Early Childhood Research Quarterly*, *36*, 79–90. https://doi.org/10.1016/j.ecresq.2015.12.009
Cowan, N. (2014). Working Memory Underpins Cognitive Development, Learning, and Education. *Educational Psychology Review*, *26*(2), 197–223. https://doi.org/10.1007/s10648-013-9246-y
Friedman, N. P., & Miyake, A. (2017). Unity and diversity of executive functions: Individual differences as a window on cognitive structure. *Cortex; a Journal Devoted to the Study of the Nervous System and Behavior*, *86*, 186–204. https://doi.org/10.1016/j.cortex.2016.04.023
Friso-van den Bos, I., van der Ven, S. H. G., Kroesbergen, E. H., & van Luit, J. E. H. (2013). Working memory and mathematics in primary school children: A meta-analysis. Educational Research Review, 10, 29–44. https://doi.org/10.1016/j.edurev.2013.05.003
Luck, S. J., & Vogel, E. K. (1997). The capacity of visual working memory for features and conjunctions. *Nature*, *390*(6657), Article 6657. https://doi.org/10.1038/36846
Meyer, D. E., & Kieras, D. E. (1997). A computational theory of executive cognitive processes and multiple-task performance: Part 2. Accounts of psychological refractory-period phenomena. *Psychological Review*, *104*(4)
Miyake, A., & Shah, P. (Eds.). (1999). Models of working memory: Mechanisms of active maintenance and executive control. Cambridge University Press. https://doi.org/10.1017/CBO9781139174909
National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Author.
Pólya, G. (1945/2014). *How to solve it: A new aspect of mathematical method*. Princeton University Press.
Lester, F. K., & Cai, J. (2016). Can Mathematical Problem Solving Be Taught? Preliminary Answers from 30 Years of Research. In P. Felmer, E. Pehkonen, & J. Kilpatrick (Eds.), Posing and Solving Mathematical Problems: Advances and New Perspectives (pp. 117–135). Springer International Publishing.
Raghubar, K. P., Barnes, M. A., & Hecht, S. A. (2010). Working memory and mathematics: A review of developmental, individual difference, and cognitive approaches. Learning and Individual Differences, 20(2), 110–122. https://doi.org/10/fs4ghp
Rhodes. S., Bryck, R., DePiro, A., Sethuraman, S. (In Preparation). The impact of web-based problem solving application on student problem solving proficiency.
Rhodes, S., Bryck, R., Gutierrez de Blume, A. (2023). Exploring factors influencing success in mathematical problem solving. *Proceedings of the 44th annual conference of the North American Chapter of the International Group for the Psychology of Mathematics Education (PME-NA).*
Schoenfeld, A. H. (2013). *Reflections on Problem Solving Theory and Practice*. *The Mathematics Enthusiast, 10* (2), 9-34. Retrieved from https://scholarworks.umt.edu/cgi/viewcontent.cgi?article=1258&context=tme
Singer F. M., Voica C. (2013). A problem-solving conceptual framework and its implications in designing problem-posing tasks. *Education Studies in Mathematics, 83*, 9-26. https://doi.org/10.1007/s10649-012-9422-x

Smith, M. S., & Stein, M. K. (2018). *5 practices for orchestrating productive mathematics discussions* (2nd ed.). National Council of Teaching of Mathematics.

Younger, J., O'Laughlin, K., Anguera, J. A., Bunge, S., Ferrer, E. E., Hoeft, F., McCandliss, B., Mishra, J., Rosenberg-Lee, M., Gazzaley, A., & Uncapher, M. (2022). Better together: Novel methods for measuring and modeling development of executive function diversity while accounting for unity. PsyArXiv. https://doi.org/10.31234/osf.io/ctzwx

Zelazo, P. D., Blair, C. B., & Willoughby, M. T. (2017). *Executive function: Implications for education (NCER 2017-2000)*. Washington, DC: National Center for Education Research, Institute of Education Sciences, U.S. Department of Education. This report is available on the Institute website at http://ies.ed.gov/.