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MATHEMATICS EDUCATION**

*Changing Mathematics Education by
Educating for Change*



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**Changing Mathematics Education by
Educating for Change**

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VALIDITY EVIDENCE FOR THE MIDDLE SCHOOL MATHEMATICAL BELIEFS SCALES (MMBS)

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Students' beliefs about themselves and about what it means to be effective learners of mathematics has been shown to influence multiple factors such as their success, engagement, and motivation (Boaler, 2016; House, 2006; Mazana et al., 2019). This paper reports evidence of validity for middle school mathematical beliefs scales (MMBS). Findings suggest that the MMBS has a solid six-factor structure that demonstrates robust reliability and construct validity.

Keywords: Affect, Emotion, Beliefs, and Attitudes; Middle School Education

The beliefs students hold about the nature of learning, mathematics, and about their own ability impact student engagement, motivation, success, and even the strategies that they use (Boaler, 2016; House, 2006; Mazana et al., 2019). Beliefs are operationalized herein as subjective, value-laden mental constructs that individuals perceive as accurate (Philipp, 2007; Skott, 2014). Researchers have categorized mathematical beliefs into beliefs about the nature of mathematics, beliefs about learning mathematics, and beliefs about one's mathematical ability (Di Martino & Zan, 2015). As few scales exist that measure beliefs across multiple categories the purpose of the present study was to develop and collect evidence of validity for the MMBS.

The MMBS includes subscales of self-efficacy, growth mindset, valuing mistakes, challenges, and perseverance, speed, using experiences in school mathematics, and mastery orientation. Self-efficacy is one's belief in their own capability to be successful in a given situation (Bandura, 1977). Self-efficacy has been found to be related to better problem-solving skills, lower math anxiety, and the increased use of strategies for self-regulation (Hoffman & Schraw, 2009; Schunk & Pajares, 2002; Usher & Pajares, 2009; Zimmerman, 2000).

Growth mindset is defined as the belief that abilities and intelligence can grow over time (Dweck, 2006). The research on growth mindset has been mixed with some studies suggesting that having a growth mindset is related to increased success, resilience, risk-taking, problem-solving abilities, and focus on learning (Blackwell et al., 2007; Boaler, 2016; Burnette et al., 2013; Claro et al., 2016; Yeager & Dweck, 2020). However, other studies have found little to no impact of growth mindset interventions (Macnamara & Burgoyne, 2023).

Valuing mistakes, challenges, and perseverance in learning, embodies a student's conviction that encountering difficulties and making mistakes is vital for growth in the learning process (Ingram et al., 2013), and has been linked to resilience (Duckworth et al., 2007), constructive failure (Kapur, 2008), and increased problem-solving abilities (Hattie & Timperley, 2007).

Research suggests solving problems quickly is correlated with academic performance (Cowan et al., 2006). Speed is frequently emphasized in classroom practices such as fluency drills, timed assessments, and standardized testing (Geary, 2011). However, an excessive focus on speed can exacerbate math anxiety and have detrimental outcomes for student learning and conceptual comprehension (Ramirez et al., 2013).

Seeing how the mathematics learned in schools connects to one's life can increase motivation (Fennema et al., 1981) and support the development of positive dispositions towards mathematics (Benson-O'Connor et al., 2019). The present study sought to operationalize this category as both the belief that school mathematics is useful in the real world as well as the belief that daily experiences can be useful in making sense of mathematical concepts.

Mastery orientation refers to an individual's intrinsic motivation to develop competence and master new skills rather than focusing on external validation or outperforming others (Dweck & Leggett, 1988). Research suggests that mastery orientation is associated with positive outcomes such as greater intrinsic motivation, deeper cognitive engagement, academic efficacy, and higher resilience when facing academic challenges (Furner & Gonzalez-DeHass, 2011; Pintrich, 2000).

The Present Study

The research question for the present study was: What is the latent factor structure and reliability of the MMBS? These scales are intended to measure the mathematical beliefs of middle school students and to explore the relationship between these beliefs and other factors. The scales should not be used to group students or predict future achievement.

Development started by reviewing existing scales and research on mindset, beliefs, and self-efficacy (e.g., Dweck, 2006; Fennema & Sherman, 1976; Kirmizi & Tarim, 2022; Kloosterman & Stage, 1992; Usher & Pajares, 2009; Vazquez et al., 2020; Watson et al., 2021). From this, 34 items were written which were then reviewed by a panel of experts in mathematics and equity. After items were revised or removed based on feedback, 29 items remained, which were then administered to 7th and 8th grade students, with 239 students having complete data. Exploratory factor analysis with a common factor extraction (principal axis factoring) with an oblique rotation (direct oblimin) was employed and items that did not load as theorized were examined and revised. New items and factors were also added resulting in a 35-item eight-factor solution.

Method

Participants and Sample

All participants were middle school students drawn from a single school located in a large, suburban district on the East Coast of the United States. The 248 students with complete data self-identified as girl (114), boy (130), non-binary (3), and 1 who did not self-report gender. There were 132 students enrolled in 7th grade and 116 in 8th grade.

Materials and Instruments

The 35-item, eight-factor, scale was administered via Qualtrics. To increase reliability (Schraw, 2009), items were given as sliding scales where respondents could select any value between 0 and 100, inclusive. The Flesh reading ease was 78.7 with a Flesh-Kincaid grade level of 5.2. Moreover, reliability coefficients, McDonald's Ω , ranged from 0.77-0.88.

Data Analysis

EQS 6.4 software (Bentler, 2005) was employed to conduct a series of standard CFA nested models. Data were tested against requisite statistical assumptions and screened for outliers prior to data analysis. The data did not contain any univariate or multivariate outliers, and it met all requisite statistical assumptions except multivariate normality. No cases were deemed univariate or multivariate outliers; thus, all 248 cases were retained for analysis. Nevertheless, data demonstrated multivariate kurtosis (Mardia's Normalized estimate was 64.34; according to Bentler, 2005, any values exceeding 6 are considered multivariate non-normal, with greater distance from 6 indicating greater degrees of multivariate non-normality), and thus, maximum likelihood robust (MLR) statistics were requested in lieu of the normal distribution statistics.

We began with a standard CFA eight-factor solution previously described followed by the more parsimonious six-factor modified solution. The models were compared using the Satorra-Bentler scaled chi-square difference test (S-B $\Delta\chi^2$) for overall best fit to the observed data. Because MLR statistics correct for multivariate non-normality, comparing the models using a non-scaled, normal distribution $\Delta\chi^2$ is inappropriate (Satorra & Bentler, 2010). Presumably, the model with the highest fit indices and lowest residuals would be a statistically significant improvement over all other models. We accounted for auto-correlation in the data by correlating the residuals of relevant manifest and latent variables, as recommended by Kline (2011) for data that are dependent, and thus, taking into consideration within-person shared variance. Pearson's r coefficients for the residuals ranged from $r = .26$ to $r = .49$, suggesting no multicollinearity among the residuals.

Goodness-of-fit indices (*NNFI, *CFI, *IFI) $\geq .90$ suggest an adequately fitting model, and those $\geq .95$ suggest excellent fit of the model to observed data. With respect to residuals, standardized root mean square residual (SRMR) values $\leq .11$ suggest reasonable errors in estimating model parameters and root mean square error of approximation (*RMSEA) values $\leq .08$ suggest that the model parameters approximate those of the population reasonably well (Byrne, 2006; Kline, 2011). *Dillon-Goldstein's rho* (ρ) was also used to assess the overall or composite reliability of the model. *Rho* measures how well the manifest/indicator variables, as a block, represent the latent variable in which they are hypothesized to load. Similar to the interpretation of Cronbach's alpha, higher values for *rho* indicate greater model reliability, with .70 serving as the lower-bound for adequate model reliability (Werts et al., 1974).

Results

Eight-Factor Initial Solution

The eight-factor solution was ill-fitting to the observed data, S-B χ^2 ($N = 248$, $df = 783$) = 1215.11, $p < .001$, *NNFI = .883, *CFI = .893; *IFI = .895, SRMR = .084, RMSEA = .047 (CI90% = .042, .052). Inspection of the standardized solution indicated that one item exhibited a negligible factor loading. In addition, the three latent variables dealing with valuing (mistakes, struggles, persistence in math) demonstrated high multicollinearity (Pearson's $r \geq .90$). Finally, review of the Lagrange Multiplier Test for adding parameters suggested that adding five error correlations related to the math growth-mindset latent variable indicators, and the valuing latent variable indicators were necessary, along with one cross-loading item. Given that all modifications were theoretically defensible, this initial model was adjusted accordingly.

Six-Factor Modified Solution

This model specified a modified six-factor structure compared to the eight-factor model, in which valuing mistakes in math, valuing struggle in math, and valuing persistence in math were all collapsed into one factor. The six-factor solution fit the observed data reasonably well, S-B χ^2 ($N = 248$, $df = 506$) = 691.98, $p < .001$, *NNFI = .940, *CFI = .946; *IFI = .947, SRMR = .063, RMSEA = .039 (CI_{90%} = .031, .045). Standardized factor loadings were all significant and ranged from .332 to .886. Factor correlations were all positive and statistically significant, ranging from $r = .28$ to $r = .79$. The cross-loading item, GM3, exhibited a standardized factor loading of .598 in its original math growth-mindset latent variable and .372 in the math self-efficacy latent variable. Correlations between the error variances of the indicators previously mentioned were all positive and significant, ranging from $r = .31$ to $r = .49$. Finally, *Dillon-Goldstein's rho* = .958, suggesting excellent composite model reliability. No other modifications made substantive theoretical sense, and hence, this was deemed the final model.

Comparison of Nested Models Employing the S-B $\Delta\chi^2$

Table 1 displays the scaling correction factor for the nested models and Table 2 contains the results of the S-B $\Delta\chi^2$. As is evident from Table 2, the S-B $\Delta\chi^2$ is statistically significant ($p < .001$), indicating that the six-factor model provides a superior statistical fit to the observed data when compared to the eight-factor model. Hence, evidence suggests that the more parsimonious six-factor solution should be retained in lieu of the more complex eight-factor solution.

Table 1: Scaling Correction Factor Values of Nested Models

Model	S-B χ^2	<i>df</i>	SCF ^a
Eight-Factor	1215.11	783	0.682
Six-Factor	691.98	506	0.658

^a Scaling correction factor.

Table 2: Satorra-Bentler Scaled χ^2 Difference Test Results Between Nested Models

Model Comparisons	CD ^a	Δdf	TRd ^b
Eight-Factor, Six-Factor	0.726	277	514.300**

Note. The model on the left is the comparison model; the model on the right is the alternative model.

^a Difference in test scaling correction

^b S-B scaled $\Delta\chi^2$ test statistic (T; TRd represents the equation used to calculate T)

** S-B scaled $\Delta\chi^2$ test is significant at $p < .001$

Discussion, Conclusion, and Limitations

Our findings suggest that the MMBS appears to be a viable option for measuring a wide range of students' beliefs about mathematics. Measuring beliefs is crucial as research indicates that students' attitudes and perceptions about mathematics can significantly impact their learning and performance (Boaler, 2016; House, 2006; Mazana et al., 2019). Furthermore, the instrument exhibited good internal consistency, suggesting that items are reliably measuring the purported mathematical belief constructs, and strengthening the validity of the obtained scores. Nevertheless, the study still had several limitations. The current study was conducted with middle school students from a single suburban middle school. Therefore, future research should validate this instrument across other contexts. Further research should also explore construct validity by examining the relationship between the scores obtained using this instrument and other relevant constructs, such as student achievement in mathematics, engagement in mathematical tasks, and teacher perceptions of student mathematical abilities. Finally, the present scale utilizes a trait-based approach to measuring student beliefs. Given that research has shown that beliefs are often impacted by state-based considerations (Middleton et al., 2017), future research should develop additional measures of state-based mathematical beliefs.

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