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Changing Mathematics Education by Educating for Change

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Changing Mathematics Education by Educating for Change

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THE IMPACT OF A WEB-BASED APPLICATION ON MATHEMATICAL PROBLEM-SOLVING PROFICIENCY IN MIDDLE SCHOOL STUDENTS

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Despite widespread calls to increase problem solving in the middle grades, little is currently known about how to best teach problem solving (Lester & Cai, 2016). This study investigated how a web-based program that combines executive function supports, metacognitive prompts, and that fosters peer interaction, affected problem-solving proficiency in a sample of 293 sixth (n = 154) and seventh (n = 139) grade students. Results show moderate effect sizes of the impacts of the intervention on students' problem-solving.

Keywords: Metacognition, Middle School Education, Problem-Solving, Technology

Introduction and Review of the Literature

Proficiency in mathematical problem solving is a critical skill within K-12 education, having vast implications for college and career readiness (Common Core State Standards Initiative, 2010; English & Gainsburg, 2016; MetLife, 2010; Murnane and Levy, 1996; National Council of Teachers of Mathematics, 2000, 2014). Research has found that teaching and learning through problem solving is related to increases in students' learning and conceptual understandings of mathematics (Boaler, 2002; Boaler & Staples, 2008; Cai, 2003; Hiebert & Wearne, 1993; Schoenfeld, 1992; Shimizu, 2003). Problem-solving experiences may also lead to increases in students' motivation and positive dispositions towards mathematics (Boaler, 2002; Cai, 2003; Stein et al., 2003).

Given the importance of problem solving, considerable research has been conducted over the past 50 years to better understand factors that influence problem solving success. Numerous studies have suggested that problem solving draws on multiple factors such as executive functions, metacognition, strategic thinking, content knowledge, and affective dispositions such as motivation and beliefs about self and subject (Chapman, 2015; Kilpatrick et al, 2001; Mayer & Wittrock, 2006; Rhodes et al., 2023; Schoenfeld, 1985, 1992; Tan & Limjap, 2018). Further exploration of each of these factors has led to robust understandings of the roles they play in the problem-solving process (e.g., Kramarski et al., 2002; Lester & Cai, 2016; Schoenfeld, 1979; Tan & Limjap 2018).

Additional research has sought to improve students' problem solving by isolating and targeting individual factors that influence problem-solving success. For example, several studies have found that using metacognitive strategies such as cognitive questioning, cognitive strategy instruction, and metacognitive training led to significant increases in students' problem-solving proficiency (Gillies et al., 2012, Kramarski et al., 2002; Montague et al., 2011; Özsoy & Ataman, 2009). Moreover, a meta-analysis conducted by Lee et al. (2018) found that the overall effect size, across 18 studies that explored the effects of metacognitive training on algebraic reasoning, was d = 0.973 with SE=0.196.

In terms of explicit instruction on heuristics, Schoenfeld (1979) found that being intentional in teaching heuristics to students was effective in increasing their problem-solving proficiency. Conversely, other studies have found little to no impact of similar interventions (e.g., Goldberg, 1974). These mixed results have led some scholars to conclude that metacognition is necessary to effectively apply heuristics when problem solving (Lesh & Zawojewski, 2007), thereby demonstrating the interconnectedness of factors influencing problem-solving success.

Further research has shown that effective problem solving requires leveraging multiple factors simultaneously—such as engaging in sense-making, metacognition, and strategic thinking (Jitendra et al., 2015; Schoenfeld, 1992), potentially suggesting that isolating factors is problematic. Despite this, few studies have sought to target multiple factors at once, and questions remain around how best to teach problem solving within K-12 classrooms (Lester & Cai, 2016).

Responding to this need to gather more conclusive evidence regarding problem-solving instruction, the present study explored the impact of an intervention designed to target multiple factors that influence problem-solving success, such as metacognition, working memory, strategic thinking, and student beliefs (Chapman, 2015; Rhodes et al., 2023; Schoenfeld, 1992). The research was driven by the following research question: Does the use of CueThinkEF+ correspond with increases in mathematical problem solving? We predicted that the experimental condition would significantly increase both the accuracy—operationalized herein as a dichotomous judgement of the correctness of a students' answer to a mathematical problem—and understanding—defined herein as a partial credit score that considered mathematical knowledge demonstrated by their work—of mathematical problem solving, when compared to the control condition ("business as usual" math instruction). We did not expect differences between grade levels, nor the experimental condition by grade level interaction to be statistically, or meaningfully, significant on either outcome.

Theoretical Framework

This work rests on two key theoretical frameworks for understanding mathematical problem solving. First, four key sets of knowledge and skills have been proposed to be successful in problem solving: resources (knowledge of math facts and procedures), heuristics (strategies and techniques to approach the problem), control (ability to monitor and regulate the process), and beliefs (mindset and attitudes towards math; Schoenfeld, 1985, 1992). Similarly, effective problem solving has been conceptualized as occurring in four key phases (Pólya, 1945/2014): understanding the problem (overview and analysis of the problem, identifying key information), planning (consider possible strategies and approaches, consider previous similar problems), solving (carry out the plan, perform necessary calculations and steps), and look back (evaluate, and reflect on the solution while considering alternative methods or room for improvement). These frameworks guided the design of the intervention which broke up the problem-solving process into phases that were aligned to the work of Pólya (1945/2014) and which supported sense-making, metacognition, positive attitudes, and strategic thinking (Schoenfeld, 1992), as described below.

Methods

Participants

Fifteen math teachers from three middle schools within a large, suburban school district on the West Coast of the United States participated in the study. Two of the schools were assigned to the control group (n = 8 teachers) with the remaining school being the intervention group (n =

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7 teachers). All students taught by the 15 participating teachers were invited to participate in the study. Within the present analyses, 293 students had complete data (*n* = 101 for the control group, *n* = 192 for the intervention group). These students' ages ranged from 10.46-15.23 years (*M* = 12.24, *SD* = 0.72). Of the 293 students with complete data, 187 identified as a girl, 102 as boy, and four as non-binary, with 138 identifying as White, 17 as Black, 18 as Middle Eastern, 113 as Hispanic, and seven as two or more races. In addition, 154 of the students were in sixth grade and 139 were in seventh grade.

Description of the Platform

The intervention involved the use of an online web-based application named CueThinkEF+. The goal of the application was to support students in learning how to solve rigorous mathematical problems. To accomplish the goal, the platform started by breaking the problem-solving process into four phases (explore, plan, solve, review) that loosely follow the phases originally theorized by Pólya (1945/2014). To start, teachers selected problems to assign to their students from a prepopulated problem bank that included problems developed by the Math Forum and Illustrative Mathematics. Some teachers also elected to write their own problems to assign to students. When students entered into the platform they were shown a thought of the day (e.g., "It's helpful to ask questions about your thinking!") that was designed to promote positive mindsets and beliefs, encourage perseverance and metacognition, or support students in transferring the scaffolds and structures that were built into the program to problem solving that was external to the program.

After selecting the assigned problem, students were brought to a main landing page that included a button for each phase of the problem-solving process noted above so that they could select which phases to use and which order to complete those phases. This structure was used to account for the fact that problem solving is rarely a linear process (Schoenfeld, 1985) and to allow students to easily return to previous phases. Thus, the order in which the phases are described below does not necessarily align to the order in which students used the phases.

Explore. The goal of the explore phase was to encourage sense-making (Schoenfeld, 1992) by asking students what they noticed and wondered about the problem (see Fetter, 2011), and by having students restate the problem in their own words. Teachers were encouraged (but not required) to facilitate the explore phase as a full class discussion to allow for discussion of different noticings and wonderings and to ensure that all students had entry points into the problem. Students also had access to various markup tools (e.g., digital highlighters), and any markups that they made were shown on the problem within all four phases.

Plan. The planning phase consisted of two main components: 1) a list of problem-solving strategies (e.g., draw a picture) that they could select from or add to; and 2) a planning journal that encouraged students to detail their plan for solving the problem and that scaffolded this process through optional sentence stems such as "If my strategy is not working, I will...". Drawing from research on metacognition and problem-solving (Kramarski, et al., 2002; Lester, 2013; Schoenfeld, 1992), the goal of this phase was to encourage students to think about what strategies might be effective, how they might apply those strategies to solve the problem, and what they could try if they get stuck.

Solve. During the solve phase, students were given a digital whiteboard that included various manipulatives and tools such as number lines, coordinate planes, tables, fraction tiles, and algebra tiles. Using these tools along with text, drawings, and/or equations, students worked to solve the problem and created a voice recording in which they explained their problem-solving process and solution to their peers. Students could also use a slide-out panel to quickly reference,

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update, or modify their planning journal while solving. This slide-out panel was added to scaffold students' working memory and to encourage them to be metacognitive by reminding them of their plan from the previous phase. This feature also allowed students to reflect on whether their plan was working or consider what other strategies they could try.

Review and Gallery. After completing their recordings, the program reminded students of what they had written in the explore phase about what the problem was asking. This feature was added to scaffold students' working memories and ensure that they were answering the original problem. Students then typed in their final answer to the problem, checked their work, and uploaded their recording and written responses within each phase to a class gallery. The gallery promoted further discussion and reflection by allowing other students to view their peers' work and to engage in conversations through annotations. The gallery also enabled teachers to play selected recordings to the class to facilitate discussions as advocated by Smith and Stein (2018).

Instruments and Scoring

Problem Solving Measure. Students' problem solving was measured using 3-item measures that were administered at pre- and post-test. For each grade level, researchers started with a bank of problem-solving items that were developed by Illustrative Mathematics (IM) for use on their assessments. These assessment items were not available within the CueThinkEF+ problem bank and are not freely available online, making it unlikely that teachers in either the intervention or control groups had access to them. This bank of problems was chosen given that most IM problems are cognitively demanding, thus ensuring that the items assessed students' problem solving rather than simply procedural knowledge. In addition, the bank was chosen because IM has a rigorous review process for their items, which included evidence of test content through standards alignment work and expert review panels, and response process through reading level analyses and equity audits. Moreover, this choice ensured that none of the items were written by the researchers.

From this bank, three distinct items were selected for each grade level. The items were selected based on three criteria:

a. The items had to be cognitively demanding and rigorous as assessed by the Smith and Stein (2018) and depth of knowledge (Webb, 2002) frameworks.
b. Only problems that aligned with standards that the district identified as high priority were selected, with one item aligning to content from each trimester. Given that the study was conducted during the COVID 19 pandemic, this choice was made to help ensure that the degree to which the topics were emphasized in classrooms was similar across classrooms and schools, and that the topics would be covered in different parts of the academic year.
c. The items had to have opportunities for student thinking to be visible through diagrams, mathematical work, or explanations.

If a problem met the first two criteria but had limited opportunities for student thinking and work to be shown, directions were added to the problem to require students to explain their reasoning. All problem-solving measures were administered on paper and pencil, with teachers being encouraged to give students as much time as needed to complete the problems, whenever possible.

Completed measures were scored for both accuracy and understanding. Problem solving accuracy was calculated by the correctness of the math problem, as determined by answer keys provided by IM. For each student, the scores for each of the three problems were added together, yielding a score between 0 and 3. Problem solving understanding was calculated by scores based on a rubric that was created with the help of an expert in mathematics education who was external to the main study. For each problem, the expert identified: 1) key elements students would need to notice and attend to as they read and unpacked the problem (e.g., "recognizing they need to find area"); and 2) how the students should mathematically attend to the key elements they noticed (e.g. "a rate stays in proportion or can be applied iteratively over time"). To establish agreement, three or four raters (depending on the grade being scored) independently evaluated ten blinded math problem solving assessments at a time, scoring them both for correctness and for understanding. Discrepancies were discussed and the rubrics were refined until sufficient agreement was reached. At that point, all remaining assessments were divided up between the raters for scoring. Fleiss' kappa for multirater agreement were done separately for each grade level and scoring type and ranged from .703 to .961–which substantively can be interpreted as "substantial agreement" to "almost perfect agreement", respectively (Landis & Koch, 1977). After scoring, understanding scores were averaged at the item-level to ensure that each question was equally weighted. These averages were then summed up, yielding a maximum possible understanding score of 3 for each test.

Research Design

The present study employed a quasi-experimental pretest/posttest mixed research design. Preexisting intact schools were selected to participate in either experimental or control groups with the final sample for the present analysis including 293 students (Treatment, n = 192; Control, n = 101). Teachers in the intervention group were provided approximately seven hours of professional learning (PL). The goal of the PLs were to train the teachers on how to use the application effectively and to develop common understandings of what problem solving is, why it is important, and how to teach it through the application. The teachers used the program approximately 5-10 times over the course of the school year. Meanwhile, teachers in the control group did not attend the PLs and continued with business-as-usual instruction.

Data Analysis

Data were screened for univariate outliers using box-and-whiskers plots and multivariate outliers using Mahalanobis Distance and Cook's D values. Data were also tested against requisite statistical assumptions including normality, multicollinearity, homogeneity of variance (univariate), homogeneity of variance-covariance matrices (multivariate), and homogeneity of regression slope coefficients. No extreme outliers were detected in the data at either the univariate or multivariate levels and data met all requisite statistical assumptions, except the homogeneity of variance-covariance matrices and the homogeneity of variance assumption for the accuracy outcome (p = .036). The understanding outcome met the assumption. As a result, Pillais' Trace, a more conservative calculation of statistics, was interpreted in lieu of Wilk's Lambda at the multivariate level. At the univariate level, robust standard errors were requested for the accuracy analysis that adjust these parameters based on the degree of violation to the homogeneity of variance.

Our research objective was met by conducting a 2 (treatment: experimental, control) x 2 (grade level: 6th, 7th) factorial multivariate analysis of covariance (MANCOVA), with pretest scores for each outcome serving as the covariates, and posttest scores as the outcomes. Partial η^2 (η^2_p) served as the measure of effect size for the MANCOVA. Cohen (1988) provided interpretive guidelines for η^2_p of: small (.010-.059); moderate (.060-.139); and large (≥ .140).

Results

Table 1 displays the descriptive statistics by group for each outcome and Tables 2 and 3 present the zero-order bivariate Pearson's product moment correlation coefficients for each group for posttest measures only.

Table 1: Descriptive Statistics for Experimental Condition and Grade Level on Math Problem Solving Understanding and Accuracy

Outcomes	Treatment		Control		6^{th} Grade		7^{th} Grade	
	M	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
Understanding	0.68	0.51	0.51	0.52	0.62	0.60	0.59	0.50
Accuracy	0.98	0.65	0.64	0.63	0.71	0.69	0.75	0.58

Table 2: Zero-Order Correlation Matrix of Posttest Outcomes for the Experimental Condition Variable

Variable	1	2
1. Understanding	-	0.54*
2. Accuracy	0.62*	-

Note. Correlations above the diagonal are for the control group and those below the diagonal are for the treatment group.
* $p < .001$ (one-tailed test of significance)

Table 3: Zero-Order Correlation Matrix of Posttest Outcomes for the Experimental Condition and Grade Level

Variable	1	2
1. Understanding	-	0.52*
2. Accuracy	0.72*	-

Note. Correlations above the diagonal are for 6th grade and those below the diagonal are for 7th grade.
* $p < .001$ (one-tailed test of significance).

Results of the 2 x 2 MANCOVA revealed that the multivariate grade level x experimental condition interaction was not significant, $p = .06$. Additionally, only the problem-solving understanding covariate significantly and meaningfully influenced the outcomes, $p < .001$, $\eta^2_p = .401$, and hence, the need to control for this variable. Only the experimental condition main effect was significant, multivariate $F(2,286) = 30.18$, $p < .001$, $\eta^2_p = .082$. Given this significant multivariate finding, the univariate results for this main effect were interpreted next.

Univariate findings for the experimental condition main effect indicated statistically and meaningfully significant results for both posttest problem solving accuracy, $F(1,287) = 24.59$, $p < .001$, $\eta^2_p = .079$, and understanding, $F(1,287) = 12.69$, $p < .001$, $\eta^2_p = .042$. Grade level exerted no significant effect on either outcome, all p-values $\geq .38$. Table 4 presents the initial means and adjusted means for both outcomes related to the experimental condition main effect.

Table 4. Initial and Adjusted Means for Significant Findings of the Effect of Experimental Condition on Math Problem Solving Understanding and Accuracy Posttest Outcomes

Outcomes	Treatment		Control	
	M	M_a	M	M_a
Understanding	0.68	0.70	0.51	0.49
Accuracy	0.98	0.99	0.64	0.62

Note. Only statistically significant findings are included for the sake of parsimony.
M = Initial (unadjusted mean); M_a = Adjusted mean, after controlling for the effect of pretest math problem solving understanding and math problem solving accuracy.
$N = 293$ (Treatment, $n = 192$; Control, $n = 101$).

Discussion and Conclusion

The present study showed that the use of CueThinkEF+—a web-based application that was designed to explicitly scaffold the problem-solving process and support students in building metacognition, strategic thinking, and productive dispositions—successfully enhanced students' math problem-solving understanding and accuracy. Results of the present study showed that using the application approximately 5-10 times over the course of a school year resulted in significant gains on students problem solving when scored as a dichotomous judgement of accuracy, $F(1,287) = 24.59$, $p < .001$, $\eta^2_p = .079$, and when scored using partial-credit rubrics that assessed problem solving and mathematical understandings demonstrated by students' work, $F(1,287) = 12.69$, $p < .001$, $\eta^2_p = .042$, when compared to students in a control group.

These findings support our hypotheses insofar as the group of middle school students exposed to the intervention significantly outperformed those in the control regarding math problem solving understanding and accuracy, albeit the observed effect was larger for accuracy than understanding. The design of the present study limits our ability to make claims about why the effect was larger for accuracy and which specific features of the product were responsible for the gains. However, given prior research has suggested that problem-solving proficiency is related to multiple factors such as metacognition, executive functions, and beliefs (Chapman, 2015; Rhodes et al., 2023; Schoenfeld, 1992), it is plausible that the observed effects were caused by a combination of factors. Prior research has also shown that metacognition plays a significant role in problem-solving success, that the relationship between metacognition and problem-solving was stronger for accuracy than understanding, and that the use of CueThinkEF+ improved students' metacognitive monitoring (Gutierrez de Blume et al., 2024; Rhodes et al., 2023). Thus, it is also possible that the features targeting metacognition (e.g., the planning journal) played a larger role in these outcomes, thereby explaining the larger effects for accuracy than understanding.

These findings support extant research on the effectiveness of problem-solving interventions that include a significant focus on metacognition (Kramarski et al., 2002), and the use of heuristics (Schoenfeld, 1979), as well as research on the critical relationship between affective factors and student success in mathematics (Goldin et al., 2016). Moreover, the present study extends prior research that has shown that success in problem-solving is related to multiple cognitive, metacognitive, and affective factors (Chapman, 2015; Rhodes et al., 2023; Schoenfeld, 1992) by demonstrating that targeting these factors simultaneously within an intervention can lead to meaningful growth in students' problem-solving proficiency.

Of practical significance, the educational intervention can be readily implemented in classrooms within existing instructional models—such as the launch-explore-summarize model or problem-solving instruction (Lappan & Phillips, 2009)—to supplement and reinforce current math instruction, and does not require additional hardware. Furthermore, the platform was tested with diverse student populations. Regarding theory, the present study extends work that has demonstrated how digital platforms can be leveraged to strengthen students' metacognition during math problem solving (Gutierrez de Blume et al., 2024).

Avenues for Future Research

While findings of this study show the utility of the educational intervention for helping middle school students with their math problem solving accuracy and understanding, additional work is needed to extend these findings to other developmental levels such as children and adults. The literature would also benefit from examining whether the findings from this study transfer across cultures, and geographically diverse areas, as math education varies by context. Moreover, given that most teachers only used the program 5-10 times over the course of a school year, additional research is needed to find the optimum amount of usage to maximize and sustain program impact. Additionally, more research is needed to better understand differences in how teachers utilize the platform within their classrooms and on any impact these differences have on student growth in problem solving. Finally, future research is needed to further isolate and understand exactly which components of the platform were the most effective drivers of the success of the platform, and how these components impacted the factors theorized to influence problem-solving success (Chapman, 2015; Schoenfeld, 1992).

Methodological Reflections and Limitations

It is important to highlight the limitations of the present work. First, the sample drawn for the present study was recruited by convenience sampling from a single school district on the West Coast, which limits the generalizability of the findings. Second, the study does not include random selection of participants to treatment versus control, as we employed a quasi-experiment versus a true experiment, which limits the internal validity of the research design.

Despite these limitations, we wish to highlight the strengths of the study. By relying on objective measures of math problem-solving understanding and accuracy rather than self-report instruments, along with a quasi-experimental research design, the confidence we have on the inferences and conclusions drawn from the findings improves. Finally, the use of a relatively robust sample size adds credence to the findings and conclusions. Thus, we think that the present study is a worthwhile contribution to the extant literature on these topics.

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One or more authors of this paper have disclosed potential financial conflicts of interest, having worked with or having an ownership stake in CueThinkEF+. These authors recused themselves from conducting the final analyses reported herein. All other authors, including those responsible for the final analyses, declare no competing financial interests.

References

Boaler, J. (2002). *Experiencing school mathematics: Traditional and reform approaches to teaching and their impact on student learning.* Erlbaum.

Boaler, J., & Staples, M. (2008). Creating mathematical futures through an equitable teaching approach: The case of Railside School. *Teachers College Record, 110*(3), 608–645.

Cai, J. (2003). What research tells us about teaching mathematics through problem solving. In F. K. Lester & R. I. Charles (Eds.), *Research and issues in teaching mathematics through problem solving* (pp. 241–254). National Council of Teachers of Mathematics.

Chapman, O. (2015). Mathematics teachers' knowledge for teaching problem solving. *International Journal on Math, Science and Technology Education, 3*(1), 19-36. https://doi.org/10.31129/lumat.v3i1.1049

Cohen, J. (1988). *Statistical power analysis for the behavioral sciences*. Routledge.

Common Core State Standards Initiative. (2010). *Common core state standards for mathematics.* National Governors Association Center for Best Practices and the Council of Chief State School Officers http://www.corestandards.org/wp-content/uploads/Math_Standards.pdf

English, L. D., & Gainsburg, J. (2016). Problem solving in a 21st-century mathematics curriculum. In L. D. English & D. Kirshner (Eds.), *Handbook of international research in mathematics education* (3rd ed., pp. 313-335). Routledge.

Fetter, A. (2011). *Ever wonder what they'd notice?* [Video]. YouTube. https://www.youtube.com/watch?v=a-Fth6sOaRA

Gillies, R. M., Nichols, K., Burgh, G., & Haynes, M. (2012). The effects of two strategic and meta-cognitive questioning approaches on children's explanatory behaviour, problem-solving, and learning during cooperative, inquiry-based science. *International Journal of Educational Research, 53*, 93-106. https://doi.org/10.1016/j.ijer.2012.02.003

Goldberg, D. J. (1974). *The effects of training in heuristic methods on the ability to write proofs in number theory* (Publication No. 7507836) [Doctoral dissertation, Columbia University]. ProQuest Dissertations & Theses Global.

Goldin, G. A., Hannula, M. S., Heyd-Metzuyanim, E., Jansen, A., Kaasila, R., Lutovac, S., Di Martino, P., Morselli, F., Middleton, J. A., Pantziara, M., & Zhang, Q. (2016). Attitudes, beliefs, motivation and identity in mathematics education: An overview of the field and future directions. ICME-13 Topical Surveys. Springer. https://doi.org/https://doi.org/10.1007/978-3-319-32811-9_1

Gutierrez de Blume, A., Rhodes, S., & Bryck, R., (2024). Improving metacognitive monitoring in math using CueThinkEF+. In K. W. Kosko, J. Caniglia, S. A. Courtney, M. Zolfaghari, & G. A. Morris (Eds.), *Proceedings of the 45th annual conference of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 1587-1592). Kent State University.

Hiebert, J., & Wearne, D. (1993). Instructional tasks, classroom discourse, and students' learning in second-grade arithmetic. *American Educational Research Journal, 30*(2), 393–425. https://doi.org/10.3102/00028312030002393

Jitendra, A. K., Petersen-Brown, S., Lein, A. E., Zaslofsky, A. F., Kunkel, A. K., Jung, P. G., & Egan, A. M. (2015). Teaching mathematical word problem solving: The quality of evidence for strategy instruction priming the problem structure. *Journal of Learning Disabilities*, *48*(1), 51–72. https://doi.org/10.1177/0022219413487408

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. National Academy Press.

Kramarski, B., Mevarech, Z. R., & Arami, M. (2002). The effects of metacognitive instruction on solving mathematical authentic tasks. *Educational Studies in Mathematics, 49,* 225–250. https://doi.org/10.1023/A:1016282811724

Landis, J. R., & Koch, G. G. (1977). The measurement of observer agreement for categorical data. *Biometrics, 33(*1), 159–174.

Lappan, G., & Phillips, E. (2009). A designer speaks: Glenda Lappan and Elizabeth Phillips. *Educational Designer, 1*(3), 1-19. https://www.educationaldesigner.org/ed/volume1/issue3/article11/pdf/ed_1_3_lappan_09.pdf

Lee, Y., Capraro, M. M., Capraro, R. M., & Bicer, A. (2018). A meta-analysis: Improvement of students' algebraic reasoning through metacognitive training. *International Education Studies, 11*(10), 42-49.

Lesh, R., & Zawojewski, J. S. (2007). Problem solving and modeling. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 763–804). Information Age.

Lester, F. K. (2013). Thoughts about research on mathematical problem—Solving instruction. *The Mathematics Enthusiast, 10*(1),245-278. https://doi.org/10.54870/1551-3440.1267

Lester, F. K., & Cai, J. (2016). Can mathematical problem solving be taught? Preliminary answers from 30 years of research. In P. Felmer, E Pehkonen, & J. Kilpatrick (Eds.), *Posing and solving mathematical problems* (pp. 117–135). https://doi.org/10.1007/978-3-319-28023-3
Mayer, R. E., & Wittrock, R. C. (2006). Problem solving. In P. A. Alexander & P. H. Winne (Eds.), *Handbook of educational psychology (2nd ed.*, pp. 287-304). Erlbaum.
MetLife. (2010). *The MetLife survey of the American teacher: Preparing students for college and careers* [Report]. ERIC. https://files.eric.ed.gov/fulltext/ED519278.pdf
Montague, M., Enders, C., & Dietz, S. (2011). Effects of cognitive strategy instruction on math problem solving of middle school students with learning disabilities. *Learning Disability Quarterly, 34*(4), 262–272.
Murnane, R., & Levy, F. (1996). *Teaching the new basic skills: Principles for educating children to thrive in a changing economy*. The Free Press.
National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.*
National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all.*
Özsoy, G., & Ataman, A. (2009). The effect of metacognitive strategy training on mathematical problem solving achievement. *International Electronic Journal of Elementary Education, 1(*2), 68–82.
Pólya, G. (1945/2014). *How to solve it: A new aspect of mathematical method.* Princeton University Press.
Rhodes, S., Bryck, R., & Gutierrez de Blume, A. (2023). Exploring factors influencing success in mathematical problem solving. In T. Lamberg & D. Moss (Eds.), *Proceedings of the 45th annual conference of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 92-102). University of Nevada, Reno.
Schoenfeld, A. H. (1979). Explicit heuristic training as a variable in problem-solving performance. *Journal for Research in Mathematics Education, 10,* 173–187.
Schoenfeld, A. (1985). *Mathematical Problem Solving.* Academic Press.
Schoenfeld, A. H. (1992). Learning to think mathematically: Sense-making in mathematics. In D. Grouws (Ed.), *Handbook for research on mathematics teaching and learning* (pp. 334–370). MacMillan.
Shimizu, Y. (2003). Problem solving as a vehicle for teaching mathematics: A Japanese perspective. In F. K. Lester (Ed.), *Teaching mathematics through problem solving: Prekindergarten - Grade 6*. National Council of Teachers of Mathematics.
Smith, M. S., & Stein, M. K. (2018). *5 Practices for Orchestrating Productive Mathematics Discussions* (2nd ed.). National Council of Teachers of Mathematics.
Stein, M.K., Boaler, J., & Silver, E. A. (2003). Teaching mathematics through problem solving: Research perspectives. In *Teaching mathematics through problem solving: Grades 6-12.* National Council of Teachers of Mathematics.
Tan, D. A., & Limjap, A. A. (2018). Filipino students use of metacognitive skills in mathematical problem solving: An emergent model. *International Journal of Development Research, 8*, 20430–20439.
Webb, N. L. (2002). Depth-of-knowledge levels for four content areas. *Language Arts, 28*(March), 1-9.