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MATHEMATICS EDUCATION**

*Changing Mathematics Education by  
Educating for Change*



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**Changing Mathematics Education by  
Educating for Change**

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## Table of Contents

Title Page .....	i
PME–NA History and Goals.....	iii
PME–NA 2024 Contributors.....	v
Preface .....	x
Table of Contents .....	xii
Section 1: Curriculum, Assessment, and Related Topics .....	1
Section 2: Equity and Justice.....	87
Section 3: Professional Development and In-Service Teacher Education .....	241
Section 4: Mathematical Knowledge for Teaching .....	444
Section 5: Mathematical Processes and Practices .....	469
Section 6: Policy, Instructional Leadership, and Teacher Educators .....	593
Section 7: Preservice Teacher Education .....	658
Section 8: Student Learning and Related Factors.....	915
Section 9: Teaching Practice and Classroom Activity .....	1112
Section 10: Technology and Learning Environment Design.....	1236
Section 11: Early Algebra, Algebraic Thinking, and Function .....	1337
Section 12: Geometry and Measurement.....	1471
Section 13: Number Concepts and Proportional Reasoning .....	1532
Section 14: Pre-Calculus, Calculus, and Higher Mathematics .....	1586
Section 15: Statistics, Probability, and Data Science .....	1703
Section 16: Working Groups and Research Colloquia .....	1768
Section 17: Plenaries.....	1885

## ELEVATING STUDENT AND TEACHER VOICES THROUGH CO-DESIGN: DEVELOPMENT AND USE OF AN INCLUSIVE R&D FRAMEWORK

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*This theoretical paper details the development process of a co-design framework. The framework was applied to the development of features within a mathematical application aimed at increasing opportunities for rich mathematical experiences and achievement in mathematical problem solving, particularly for students who are Black, Latinx (or Latino), or experiencing poverty. The framework aims to position all stakeholders as experts in the design process.*

**Keywords:** Problem-Solving; Equity, Inclusion, Diversity

Traditional research structures often draw sharp divides between researcher and participant roles which can result in negative outcomes for the participating communities due to a lack of trust, a sense of feeling like a specimen, and a sense of exploitation can inhibit the possibility of positive gains that research can provide to communities (Chicago Beyond, 2018). To combat this, some researchers and educational designers have employed co-design to center participants in studies as vital members of the development process whose experiences and perspectives are valued as core to the central mission of the research (McKercher, 2020).

Co-design is operationalized herein as “a highly-facilitated, team-based process in which teachers, researchers, and developers work together in defined roles to design an educational innovation, realize the design in one or more prototypes, and evaluate each prototype’s significance for addressing a concrete educational need” (Roschelle et al., 2006, p. 606). Within the present paper, this definition was also expanded to include students. The use of co-design has been found to lead to increased user satisfaction and system quality, better understanding of the needs of users, increased alignment between curricula and educational technologies, increased student engagement, and increased potential for consideration of non-traditional solutions (Kujala, 2003; McKercher, 2020; Roschelle et al., 2006; Steen et al., 2011). Given these benefits, the purpose of the present paper is to propose a theoretical co-design framework and discuss the application of this framework in the redesign of an online math program called CueThinkEF+

### Descriptions of Collaborators

Students and teachers who collaborated in co-design were drawn from two different districts, with each district participating in co-design for two years. Co-design students were nominated by teachers who were asked to select students of varied prior achievement from historically underrepresented backgrounds. For teachers, anyone who participated in the larger study was also invited to participate. Each year, new students were selected for the co-design groups, but two of the year 2 teachers also participated in co-design in year 1, and three of the year 4 co-

design teachers participated during year 3. Teacher and student co-design sessions were held separately to ensure everyone felt safe sharing. See Table 1 below for more information.

**Table 1: Information About Co-Design Collaborators by Year**

Year	District	Number of Co-Design Students (Sessions)	Number of Co-Design Teachers (Sessions)
1	Large Suburban District on the West Coast	10 (6)	2 (7)
2	Large Suburban District on the West Coast	8 (5)	5 (7)
3	Large Suburban District on the East Coast	14 (2)	5 (1)
4	Large Suburban District on the East Coast	6 (3)	6 (3)

### **Development Process**

Using prior research on co-design, we created the first version of the framework, which was piloted for one year. Reflection on this pilot with advisors prompted a significant redesign to the framework, which was then used for three additional years.

### **Elements of Co-design**

To develop the framework, we first identified foundational elements that research had suggested may enhance the quality of the collaboration within co-design groups. One key distinction noted in recent research is the difference between designing with, instead of for, collaborators. Benz et al. (2024) define the former as ensuring the diverse experiences, views, and assets of stakeholders are utilized throughout the design process. This contrasts with the latter in which collaborators' roles are confined to providing feedback or evaluation of products that were designed with limited stakeholder input. Hart's (1992) framework, based on the idea of citizenship participation from Arnstein's (1969) Ladder of Citizen Participation, concisely captures layers of stakeholder involvement by displaying collaboration on a continuum between manipulation and authentic shared decision making. Moreover, we noted critical co-design elements of creating safe spaces and ensuring that participants have the time and space to reflect and think critically as both individuals and as a group, often considering past experiences, present circumstances, and future goals (Vargas et al., 2022; Zamenopoulos & Alexiou, 2018).

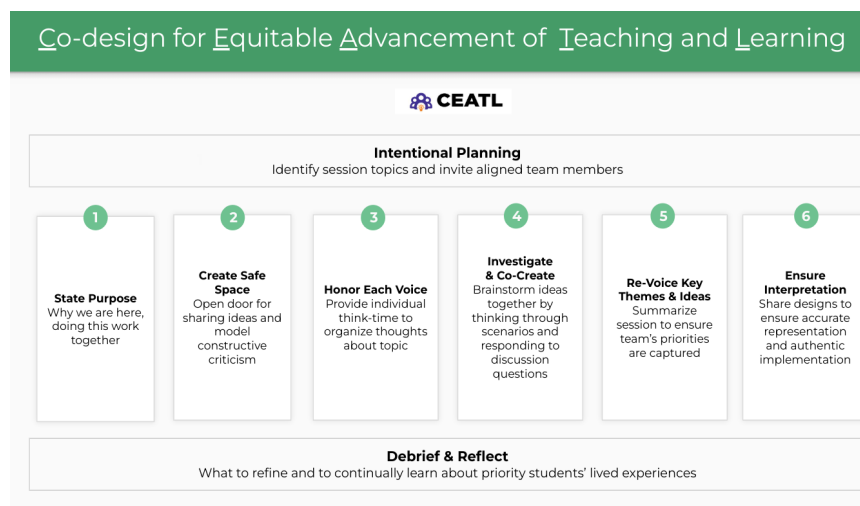
### **Faux-Design**

The goal of the first version of our co-design framework was to incorporate many of the elements noted above to create an iterative process of product design. The resultant phases were: 1) Plan Session; 2) Create Safe Space; 3) Model Feedback; 4) Present Designs; 5) Solicit Feedback; and 6) Iterate on Designs. Although full explication of this initial framework is beyond the scope of the present paper, it is important to note that students and teachers appeared to gain more comfort with sharing their feedback, and some voiced that they felt heard when they saw changes in the app as a result of their input. However, given that we brought designs to co-design that had previously been worked on by members of the team, we realized that it was more a feedback cycle/loop (faux-design) or adult-initiated with shared decisions from students (Hart, 1992; Treseder, 1997). Consequently, we redesigned the framework to elicit student and teacher ideas without influencing them with pre-created mockup designs.

## CEATL Framework

The revised framework, Co-design for Equitable Advancement of Teaching and Learning, CEATL, sought to increase the focus on co-creation and revoicing key ideas (see Fig. 1).

**Figure 1: The CEATL Framework**



In *State the Purpose*, we initiated the problem, stated the session goals, and highlighted the broader impact of the work. Co-designers were positioned as experts, and all responses were affirmed to catalyze more input. We then sought to *Create a Safe Space* for our co-design partners by ensuring everyone was included and felt safe to learn, contribute, and challenge. To do this, we showed vulnerability in our own thinking and showed that we were not holders of the knowledge through wonderings like “How do you envision it could look?” We also created norms (e.g., “it’s all on the table”) to guide productive collaboration and foster a safe space. *Honor Each Voice*, let co-designers know that everything was on the table and that, as experts, they could question everything, suggest any change they wanted, or brainstorm entirely new aspects to the platform. Next, the co-design groups collaborated to *Investigate and Co-create Ideas*. Once a central question was brought to the co-designers, we would put up a digital comment board to capture ideas. The open structure of this allowed co-designers to express ideas anonymously with no risk or threat of being criticized or judged for their thoughts. Moreover, each response was read, appreciated, and affirmed as a positive contribution for what it could bring to the platform. We then *Revoiced Key Themes and Ideas* to member-check our interpretations which were then used to revamp the platform. After redesign, we *Ensured Accurate Interpretation* of their ideas in terms of how they were represented in the product and allowed co-designers to provide additional feedback. They were often excited to see how their opinion and perspective were mechanisms for transformation in this national product.

### Piloting of the CEATL Framework with a Math Application

As part of the framework design process, co-design sessions around redesigning the CueThinkEF+ application were conducted. Figures 2 and 3 show images of co-designed features. For Figure 2, codesign students highlighted the importance of receiving specific and helpful peer feedback, saying things like: “It’s not very helpful if [my peers] say ‘I liked it’ or ‘I

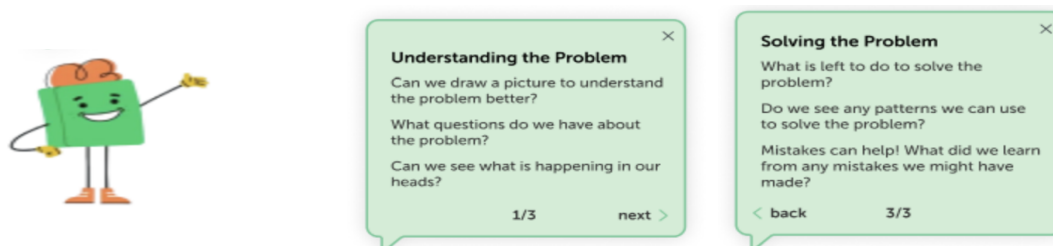


didn't like it", and "I think it is good when they explain what they liked or didn't like." Subsequent co-design discussions led to the development of an AI-enhanced annotation helper that draws from students' prior annotations to provide suggestions to students on how they can write more robust annotations. Other changes brought about or enhanced by our sessions with students and teachers included a re-designed landing page to help students navigate through problems in non-linear ways, a more structured resource bank for teachers, and a metacognitive helper (see Figure 3) that pops up to provide in-the-moment metacognitive training and support to students. After going through the co-design process, the new and redesigned features in the application not only reflected student ideas but also demonstrated better alignment with research on problem solving which has shown that student thinking is often non-linear and dynamic, that metacognition plays an important role in problem solving, and the potential benefits from creating opportunities for productive student discourse (Goos et al., 2002; Schoenfeld, 1992).

**Figure 2: Before and After from a Student Co-design Session**



**Figure 3: A Metacognitive Helper Developed Through Co-design**



### Discussion and Avenues for Future Research

This new framework allowed teachers and students to have authentic contributions to changing the platform to improve its use. Moving forward, the CEATL framework has the potential to be used in numerous settings to authentically incorporate the perspectives and lived experiences of users. Further research can explore how participants reacted to the co-design process, consider additional improvements, and can apply the framework to future research.

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